ASP4000 Stars

Homework Set 2

Perform all calculations and provide results using cgs units or solar units where appropriate. Final results for long time scales may be converted to "yr".

1. **Crystallisation**

   Consider a gas of $^{57}\mathsf{Fe}$ at a temperature of $10^9\,\mathsf{K}$.

   (a) **Estimate at what density will it transition from gas to liquid ($\Gamma = 1$) and at what density it will transition from liquid to crystal ($\Gamma = 180$).** Use Wiegner-Seitz radius to estimate characteristic distance between nuclei.

   [6 marks]

   (b) **For each density, estimate the Fermi Energy of the electron gas. Is the electron gas degenerate? Is the iron Ion gas degenerate? Please argue your case.**

   [8 marks]

   (c) **Would you expect electron captures on the iron to occur at either of these densities? Why?**

   [4 marks]

2. **Equation of State.**

   Consider a photon gas with energy density $U = aT^4$ and pressure $P = \frac{a}{3}T^4$.

   Using the first and second laws of thermodynamics, $\mathsf{d}u = T\mathsf{d}s - P\mathsf{d}v$, derive the adiabatic index for such a photon gas.

   NOTE: Here u is the specific energy, $v = 1/\rho$ is the specific density, s is the specific entropy, a is the radiation constant, and T is the temperature.

   [6 marks]

3. **Neutrinos**

   Consider the following neutrino fluxes and energies from the sun:

Source		Flux at Earth ($m^{-2}\,s^{-1}$)	Energy (MeV)	Average (MeV)
$p + p$	$\longmapsto {}^2H + e^+ + \nu_e$	6.0×10^{14}	≤ 0.42	0.263
${}^7Be + e^-$	$\longmapsto {}^7Li + \nu_e$	4.9×10^{13}	0.86 (90 %); 0.38 (10 %)	0.80
${}^8B$	$\longmapsto {}^8Be + e^+ + \nu_e$	5.7×10^{10}	≤ 15	7.2

(a) **What is the energy flux ($erg\,cm^{-2}\,s^{-1}$) at the surface of the earth?**

[2 marks]

(b) **How many solar neutrinos are on average in a box of $1\,cm^3$ on the surface of the earth at any given time?**
Assume neutrinos move at the speed of light.

[2 marks]

(c) **What is the energy density from neutrinos (in erg/cm^3) at the surface of the earth?**

[2 marks]

4. **Nuclear Reaction Rates**
Based on the general dependence of a non-resonant binary nuclear reaction,

$$\langle\sigma v\rangle \propto (k_B T)^{-2/3} \exp\left\{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m_{\rm red}}{k_B T}\right)^{1/3}\right\}$$

compute the temperature sensitivity of one of the key neutron source reactions, ${}^{22}Ne(\alpha,n){}^{25}Mg$ at $T = 3\times10^8\,K$, that is, compute the exponent n in

$$\langle\sigma v\rangle \propto T^n$$

where n is given by

$$n = \frac{d\ln\langle\sigma v\rangle}{d\ln T}\ .$$

This should be done analytically. Here Z_i are the charges of the nuclei, e is elementary charge, h is the Planck constant, k_B the Boltzmann constant, and $m_{\rm red}$ the reduced mass of the two nuclei.

[8 marks]

5. **Stellar Collapse.**

Assume a star initially in hydrostatic equilibrium collapses to a black hole. For simplicity, let's assume each shell collapses to the center in the free-fall time scale (dynamical time scale) given by

$$\tau_{\rm ff} = \sqrt{\frac{3\pi}{32G\bar{\rho}(m)}}$$

where $\bar{\rho}(m)$ is the average density inside mass coordinate m, and we neglect general relativity and pressure *during the collapse* (but not for the initial configuration of the star; "dust collapse"; Kippenhanh & Weigert, 1990, Eq. 27.10).

Compute the mass accretion rate onto the central black hole as a function of mass coordinate, m, free fall time, τ_{ff}, density $\rho(m)$, and average enclosed density, $\bar{\rho}(m)$.
(At what accretion rate would the shell of the star at mass coordinate m accrete onto the central black hole?)

[8 marks]

6. **Stellar Evolution Project**

   Get and install the MESA stellar evolution code
   http://mesa.sourceforge.net/

   The code uses `gfortran` (Linux, MacOS).

   Follow the installation instruction at http://mesa.sourceforge.net/prereqs.html including install of the SDK and compile the code.

   Try to run the examples at http://mesa.sourceforge.net/starting.html.

   Submit a plot of a test run you made, e.g., HRD for the main sequence of a $15\,\mathsf{M}_\odot$ star of solar composition.

[8 marks]