MONASH University

School of Physics and Astronomy
ASP4000 – 2017
Due: before Class 10

ASP4000 Stars

Homework Set 3

Perform all calculations and provide results using cgs units or solar units where appropriate. Final results for long time scales may be converted to "yr".

1. **Simple Stellar Models**

   Write a program to solve the Lane-Emden Equation.

   Suggested steps are:

   (a) Convert Lane-Emden Equation into a system of 1st order linear differential equations.

   [2 marks]

   (b) find inner boundary condition as needed for integration; formally the term is 0/0, so you have to find the right limit by Taylor expansion or other method.

   $$\lim_{\xi\to 0}\frac{\mathrm{d}^2\theta(\xi)}{\mathrm{d}\xi^2}=\ldots$$

   [2 marks]

   (c) Write integration program using finite steps. Don't just use pre-compiled solver package. It is reasonably possible to do this just with a spreadsheet like Google Documents (which is what I did for a proof of concept). Obviously, you are most welcome to use Python or another common programming language (FORTRAN, C, C++) instead.

   [2 marks]

   (d) Find value of ξ where θ crosses 0. In the lecture we called this value ξ_1. If you use finite steps you could use interpolation to refine the estimate. Also obtain

   $$\left.\frac{\mathrm{d}\theta(\xi)}{\mathrm{d}\xi}\right|_{\xi_1}\,.$$

   [2 marks]

   Show and document you solution steps and program (submit program or formulas used in spreadsheet).

   Tasks:

(a) Compute a model for $n = 1.5$, $M = 1\,\mathsf{M}_\odot$, central density $\rho_\mathsf{c} = 160\,\mathsf{g\,cm^{-3}}$. Use ideal gas with $\mu = 1.3$.

What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star?

[8 marks]

(b) Compute a model for $n = 1.5$, $M = 15\,\mathsf{M}_\odot$, $\rho_\mathsf{c} = 6\,\mathsf{g\,cm^{-3}}$, and a mixture of 70 % hydrogen (^{1}H), 28 % helium (^{4}He) and 2 % of nitrogen (^{14}N), all percentages by mass fraction. Use ideal gas *with radiation*. Assume the gas is fully ionized.

What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star? What is fraction of gas to total pressure,

$$\beta = \frac{P_\mathrm{gas}}{P}$$

at the center of the star?

[8 marks]

(c) **Compare your solar model from above with key data of the actual sun. What might be reasons for the discrepancy?**

[2 marks]

2. **Mass Excess and Binding Energy.**

A table of mass excess is given at `http://2sn.org/Class/ASP4000-Stars-2017-S1/homework/mass.mas03` and on Moodle.

The table lists in each line Z, A, and mass excess per nucleus in keV.

(a) **Which is the most tightly bound nucleus (largest binding energy per nucleon) and which which is the most stable nucleus (most negative mass excess *per nucleon*)?**

[4 marks]

(b) **Why do the differences in highest binding energy and lowest mass excess arise, and what is their nuclear physics origin and implication?**

[4 marks]

(c) **In the table $^{3}\mathsf{Li}$ has negative BE - what does this mean?**

[2 marks]

3. **Stellar Evolution Project**

   Using the installed the MESA stellar evolution code from the last assignment, now run a $15\,\mathsf{M}_\odot$ star of solar composition to the end of central helium burning, until all helium is gone, e.g., to a central temperature of $\sim 5{\times}10^8\,\mathsf{K}$ or central helium abundance is $Y_\mathrm{c} \ll 1$.

   Submit a T_c-ρ_c-diagram, i.e., plot of the central temperature as a function of central density as the star evolved, i.e., a parametric plot $T(t)$ as a function of $\rho(t)$ along with an HRD diagram.

   [8 marks]