ASP4000 Stars

Homework Set 4

Perform all calculations and provide results using cgs units or solar units where appropriate. Final results for long time scales may be converted to "yr".

1. **Stellar Model with Thermonuclear Burning**

Based on the Lane Emden Equation solver developed for Homework Set 3, construct stellar models that include nuclear burning.

Assume a composition that is a mixture of 70 % hydrogen ($^{1}\mathrm{H}$), 28 % helium ($^{4}\mathrm{He}$) and 2 % of nitrogen ($^{14}\mathrm{N}$), by mass fraction.

(a) Compute a model with $n = 1.5$ and $M = 1\,\mathrm{M}_\odot$. Assume the star generates its energy according to the pp chains. Use formula 18.63 from Kippenhahn, Weigert, & Weiss (2012). For simplicity, assume $\psi = 1$ and $f_{11} = 2$. Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

Find central density such that the luminosity of star equal the solar luminosity. What is the central temperature of the star? Plot specific nuclear energy generation rate and luminosity as a function of mass coordinate. What is the radius of the star? Why does it deviate from that of the sun?

[6 marks]

(b) Compute a model with $n = 3$ and $M = 10,000\,\mathrm{M}_\odot$. Assume the star generates its energy according to the CNO cycle. Use formula 18.65 from Kippenhahn, Weigert, & Weiss (2012). Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

What is the luminosity, radius, and central density of the star for a central temperature of $2 \times 10^{7}\,\mathrm{K}$, $2.5 \times 10^{7}\,\mathrm{K}$, $3 \times 10^{7}\,\mathrm{K}$, $3.5 \times 10^{7}\,\mathrm{K}$, and $4 \times 10^{7}\,\mathrm{K}$? Please also plot temperature and density as a function of enclosed mass. Please provide results for luminosity and radius in solar units. Consider using an appropriate equation of state.

[10 marks]

2. **Nuclear Reaction Kinematics - The SnSbTe Cycle**

The tin-antimony-tellurium (Sn-Sb-Te) cycle has been identified as the end point of the so-called *rp*-process. It is a cycle very similar to the well-known CN/ON cycle(s) in that it burns four $^{1}\mathsf{H}$ into one $^{4}\mathsf{He}$. It involves the nuclei $^{103}\mathsf{In}$, $^{104}\mathsf{In}$, $^{103}\mathsf{Sn}$, $^{104}\mathsf{Sn}$, $^{105}\mathsf{Sn}$, $^{106}\mathsf{Sb}$, $^{107}\mathsf{Te}$, proton captures, i.e., (p,γ), β^{+} decays, and, in variation to the CN/ON cycles, an (γ,α) reaction on $^{107}\mathsf{Te}$.

Based on the general formula for nuclear reactions,

$$\frac{\partial}{\partial t}Y_i = \sum_{\substack{\alpha_1,\alpha_2,\ldots \\ \beta_1,\beta_2,\ldots}} \lambda_{\alpha_1\mathbf{1}+\alpha_2\mathbf{2}+\ldots\to\beta_1\mathbf{1}+\beta_2\mathbf{2}+\ldots}\frac{\beta_i-\alpha_i}{\alpha_1!\alpha_2!\ldots}Y_1^{\alpha_1}Y_2^{\alpha_2}\ldots$$

(a) **Write down the system of equations for the changes of all species in the SnSbTe cycle.** You may also want to draw these nuclei in a chart of nuclei to guide your work.

[10 marks]

(b) **Assuming steady state (also for the rest of this problem), what are the changes of each of the cycle nuclei ($Z > 2$) with respect to time?**

[2 marks]

(c) Find the half life of two radioactive nuclei in the cycle. As a characteristic of the hot cycle, these radioactive decays are by far the slowest processes in the cycle. Assume an initial composition of 70 % $^{1}\mathsf{H}$, 28 % $^{4}\mathsf{He}$, 2 % $^{104}\mathsf{In}$ by *mass fraction*. **How long does it take to burn all $^{1}\mathsf{H}$ to $^{4}\mathsf{He}$?** Derive and show first an analytic expression, then add values and compute the numerical result.

[12 marks]

(d) **Using the assumptions from above, after all the hydrogen has burnt and all radioactive decays have completed, what is the mass fraction of each of the relevant species?** Consider decays of all species to stable nuclei *after* the burning has completed.

[8 marks]