RADIATION

PLANCK: $n(\nu)\,d\nu = \dfrac{8\pi\nu^2}{c^3} \dfrac{d\nu}{e^{\frac{h\nu}{k_B T}} - 1}$

Pressure: $P = \dfrac{1}{3} \displaystyle\int_0^\infty c \dfrac{h\nu}{c} n(\nu)\,d\nu = \dfrac{1}{3} a T^4$

Energy density (recall: $u = \dfrac{1}{g} \displaystyle\int_0^\infty n(p)\,\mathcal{E}(p)\,dp$

rel Energy: $\mathcal{E} = mc^2 \left[\left(1 + \dfrac{p^2}{m^2c^2}\right)^{1/2} - 1 \right]$

limit $p \ll c$: $\rightarrow p^2/2m$

$p \gg c$: $\longrightarrow p \cdot c$

or: $dp = \dfrac{h}{c}\,d\nu$

$$n(\nu)\,d\nu = n(p)\,dp = \frac{8\pi p^2}{h^3} \frac{dp}{e^{\frac{p \cdot c}{k_B T}} - 1}$$

photon: $\mathcal{E} = h\nu$, $p = \dfrac{h\nu}{c}$

$$e = \frac{1}{3} \int_0^\infty h\nu\, n(\nu)\,d\nu = \frac{1}{g} a T^4$$

NEED: U and for NR IDEAL CASE

Q: rel IDEAL CASE!

Summary:

NR GAS: $u_{gas} = \frac{3}{2} \frac{P_{GAS}}{\rho}$

REL GAS: $u_{gas} = 3 \cdot \frac{P_{gas}}{\rho}$

RADIATION: $u_{RAD} = 3 \cdot \frac{P_{GAS}}{\rho}$

→ Rel gas & RAD behave the same!

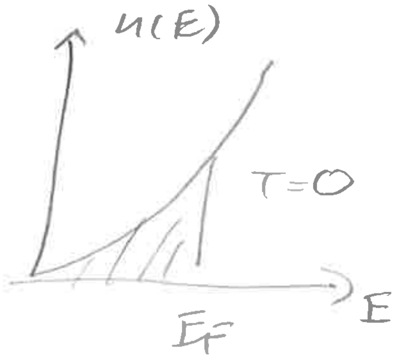
Q: What happens @ very high T?

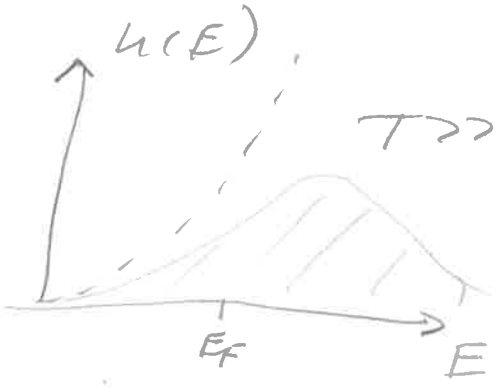
Q: where are the different regimes important?

Q: derive summary formula for Ion + e⁻ non-rel $\mu^{-1} = \sum (Z_i + 1) \cdot X_i$ for composition Z_i, X_i

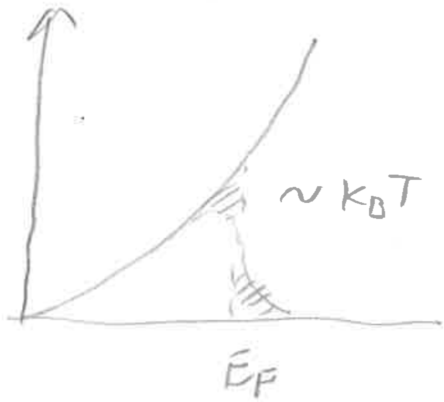
Q: Do we deal with degenerate nuclei if so — where — or why usually not? (physics...)

EXAMPLE $\boxed{\text{DEG GAS}}$





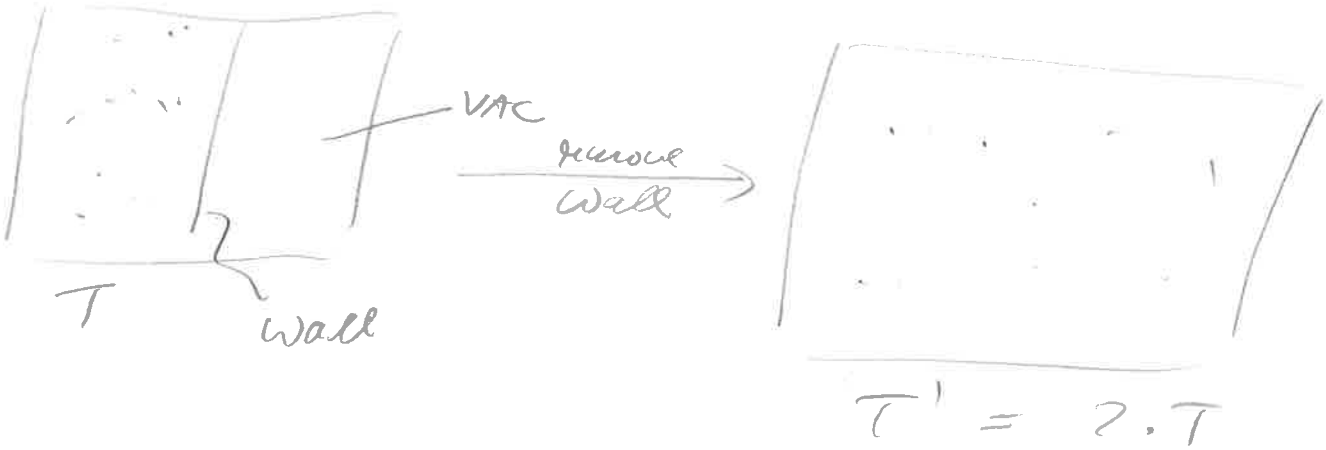
$T \gg E_F/k_B T$



finite T

$0 < T \lesssim \frac{E_F}{k_B T}$

Q: For Ideal Gas



For DEG GAS

ADIABATIC EXPONENT

$$du = T\,ds - P\,dv$$

$v = \frac{1}{\rho}$ specific volume

s : specific entropy

Q: what does "adiabatic" mean?

$$0 = du + P\,d(1/\rho)$$

in "simple" systems as discussed so far: $u \sim P/\rho$

$$\rightarrow u = \phi \cdot P/\rho$$

$$du = \phi \frac{1}{\rho} dP + \phi \cdot P\,d\left(\frac{1}{\rho}\right)$$

$$0 = \phi \frac{1}{\rho} dP + \phi\, P\,d\left(\frac{1}{\rho}\right) + P\,d\left(\frac{1}{\rho}\right)$$

$$= \phi\left(\frac{1}{\rho}\right) dP + (1+\phi)\, P\,d\left(\frac{1}{\rho}\right) \quad \Big/ \; d\frac{1}{\rho} = -\frac{1}{\rho^2} d\rho$$

$$\phi \cdot \left(\frac{1}{\rho}\right) = (1+\phi) \frac{1}{\rho^2} P\,d\rho$$

$$\frac{\phi+1}{\phi} = \frac{\rho\,dP}{P\,d\rho} = \frac{d\ln P}{d\ln \rho}\bigg|_{ad} = \gamma_{ad}$$

Q: Compute adiabatic index for:

- Ideal gas
- RAD GAS
- DEG GAS
- REL DEG GAS

Q: why do we care?

Other exponents:

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \left.\frac{d \ln P}{d \ln T}\right|_{ad} = \frac{1}{\nabla_{ad}}$$ adiabatic T gradient

$$\Gamma_3 = \left.\frac{d \ln T}{d \ln \rho}\right|_{ad} + 1$$

Q: are these all independent? why?

$$\frac{\Gamma_1}{\Gamma_3 - 1} = \frac{\Gamma_2}{\Gamma_2 - 1}$$

Crystallisation

20170303 -6-

Compare Coulomb energy to thermal E

thermal $k_B T$

Coulomb: $\frac{q^2}{d}$; $q = Z \cdot e$

d? Wigner-Seitz-Radius

$$\frac{4\pi r^3}{3} = \frac{A \cdot m_u}{\rho} \qquad d \simeq r$$

DEF Coulomb parameter $\Gamma = \frac{Z^2 e^2}{d\, k_B T}$

$\Gamma \sim$ few (1..5) gas $\Leftrightarrow$ liquid

$\sim$ 180...185 liquid $\Leftrightarrow$ solid

cf Pair correlation function

Q: What happens in mixtures?
(consider alloys!)