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Nuclei & Nuclear Reactions

MASS EXCESS

$$\Delta M = c^2 \left(m - A \cdot m_u \right)$$ unit: E!

for <u>neutral atom</u>

[Q: What is order of magnitude of differences?]

BINDING ENERGY

$$A = N + Z$$

$$E_B = c^2 \left(m - \left(N \cdot m_n + Z \left(m_p + m_e \right) \right) \right)$$

Q: What is the difference, and <u>why</u>?

Q: What are typical values?

Thermonuclear Reactions

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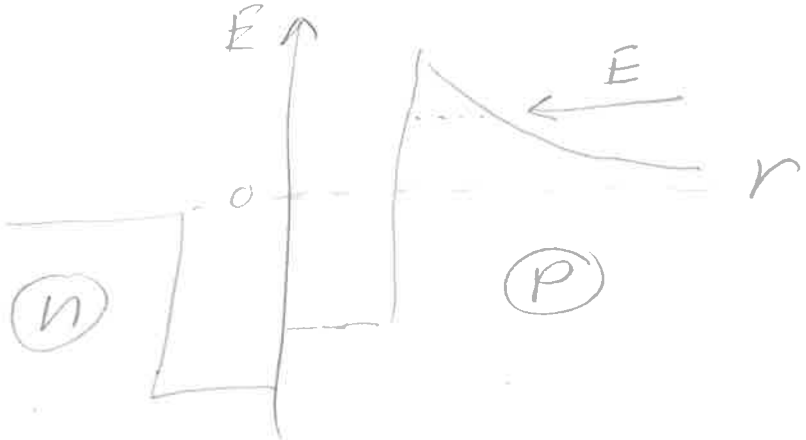
(very brief)



typical $E = k_B T \ll E_B$

Q: so: how can we have a reaction?

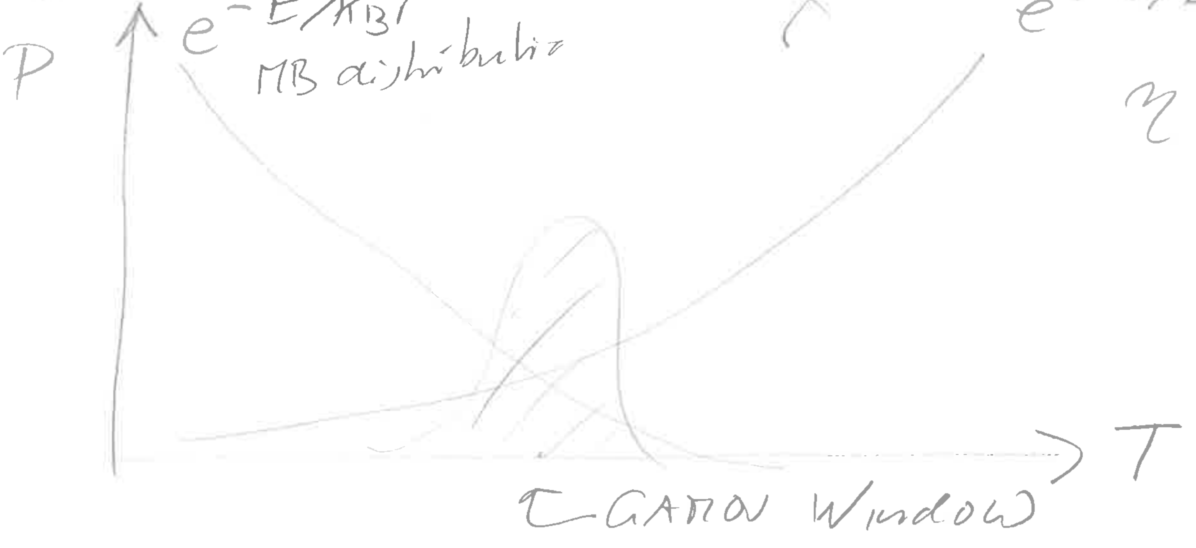
- high-E tail of distribution
- tunneling (QM)

NOTE: SCREENING @ high ρ
↳ reduce repulsive potential

Qualitative

$e^{-E/k_B T}$ MB distribution $\times$ $e^{-\eta/E^{1/2}}$ tunnel barrier penetration

$\eta = \frac{\pi}{\sqrt{2m}} \frac{Z_1 Z_2 e^2}{\hbar}$



↳ GAMOW Window

→ compute reaction rate $\langle \sigma v \rangle$

Reactions are often parametrised using an "astrophysical S-factor"

$$S(E) = \sigma(E) \cdot E \cdot e^{2\pi\eta}$$

such that

$$N_A \cdot \langle \sigma v \rangle = \left(\frac{8}{\pi m}\right)^{1/2} \frac{N_A}{(k_B T)^{3/2}} \int_0^\infty e^{-2\pi\eta} S(E) \cdot e^{-E/k_B T} dE$$

$$\simeq \left(\frac{8}{\pi m}\right)^{1/2} \frac{N_A}{k_B T} S_0 \int_0^\infty e^{-2\pi\eta - E/k_B T} dE$$

where S_0 is the S-factor in centre of Gamov Window

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NOTES ON Nuclear reactions

$$\frac{\partial}{\partial t}Y_i = \sum_{\text{reactions}} \lambda_{\underbrace{\underbrace{\alpha_1 \mathbb{1} + \alpha_2 \mathbb{2} + \dots}_{\text{IN}} \longrightarrow \underbrace{\beta_1 \mathbb{1} + \beta_2 \mathbb{2} + \dots}_{\text{OUT}}}_{\substack{\text{channel} \\ \text{INDEX OF rate}}}} \frac{\beta_i - \alpha_i}{\prod_j (\alpha_j!)} \prod_j Y_j^{\alpha_j}$$

Example

$${}^{12}C + {}^{12}C \longrightarrow {}^{23}Na + p$$

$$\lambda \underset{\substack{\uparrow \\ \alpha_1}}{2} \underset{\substack{\uparrow \\ \mathbb{1}}}{{}^{12}C} \longrightarrow \underset{\substack{\uparrow \\ \beta_2}}{1} \underset{\substack{\uparrow \\ \mathbb{2}}}{{}^{23}Na} + \underset{\substack{\uparrow \\ \beta_3}}{1} \underset{\substack{\uparrow \\ \mathbb{3}}}{p}$$

$$\frac{\partial}{\partial t} Y_{{}^{12}C} = \lambda_{2\,{}^{12}C \to {}^{23}Na + p} \frac{0-2}{2!} Y_{{}^{12}C}^2$$

$$= -\lambda_{{}^{12}C \to {}^{23}Na + p} \cdot Y_{{}^{12}C}^2$$

$$\partial t_{{}^{23}Na} = \lambda_{2\,{}^{12}C \to {}^{23}Na + p} \cdot \frac{1}{2} Y_{{}^{12}C}^2 = \frac{\partial}{\partial t} Y_p$$

if we have several reactions that change species, *all* contributions have to be added!