ASP4000 Stars

Worksheet 1

Perform all calculations and provide results using cgs units or solar units where appropriate. Final results for long time scales may be converted to "yr".

Nuclear Reaction Kinematics

Based on the general formula for nuclear reactions,

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \dots \\ \beta_1, \beta_2, \dots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \dots \to \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \dots} \frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \dots} Y_1^{\alpha_1} Y_2^{\alpha_2} \dots$$

1. **Write the equation for the change of hydrogen, $\frac{\partial}{\partial t} Y_{^1\mathrm{H}}$, in the reaction of the last step of the CNO-1 cycle for hydrogen bunring, $^{15}\mathrm{N} + {}^{1}\mathrm{H} \longmapsto {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$ (simple binary reaction prototype).**

$$\frac{\partial}{\partial t} Y_{^1\mathrm{H}} = -\lambda_{^1\mathrm{H} + ^{15}\mathrm{N} \to ^{12}\mathrm{C} + ^4\mathrm{He}} Y_{^1\mathrm{H}} Y_{^{15}\mathrm{N}}$$

2. **The system of equations for the changes $\frac{\partial}{\partial t} Y_i$ of ($i =$) ^{1}H, ^{2}H, ^{3}He, and ^{4}He due to the pp1 chain for hydrogen burning. Assume a net production of 1 (one) ^{4}He nucleus from 4 (four) ^{1}H nuclei. Assume the reaction is in equilibrium (steady state).**

$$\begin{aligned}
\frac{\partial}{\partial t} Y_{^1\mathrm{H}} &= -\lambda_{2\,^1\mathrm{H} \to ^2\mathrm{H}} \left(Y_{^1\mathrm{H}}\right)^2 - \lambda_{^1\mathrm{H} + ^2\mathrm{H} \to ^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} + \lambda_{2\,^3\mathrm{He} \to ^4\mathrm{He} + 2\,^1\mathrm{H}} \left(Y_{^3\mathrm{He}}\right)^2 && (1) \\
\frac{\partial}{\partial t} Y_{^2\mathrm{H}} &= +\lambda_{2\,^1\mathrm{H} \to ^2\mathrm{H}} \frac{1}{2} \left(Y_{^1\mathrm{H}}\right)^2 - \lambda_{^1\mathrm{H} + ^2\mathrm{H} \to ^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} && (2) \\
\frac{\partial}{\partial t} Y_{^3\mathrm{He}} &= +\lambda_{^1\mathrm{H} + ^2\mathrm{H} \to ^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} - \lambda_{2\,^3\mathrm{He} \to ^4\mathrm{He} + 2\,^1\mathrm{H}} \left(Y_{^3\mathrm{He}}\right)^2 && (3) \\
\frac{\partial}{\partial t} Y_{^4\mathrm{He}} &= +\lambda_{2\,^3\mathrm{He} \to ^4\mathrm{He} + 2\,^1\mathrm{H}} \frac{1}{2} \left(Y_{^3\mathrm{He}}\right)^2 && (4)
\end{aligned}$$

3. **Assuming steady state, what are the timely changes of ^{2}H and ^{3}He?**

$$\frac{\partial}{\partial t} Y_{^2\mathrm{H}} = 0 = \frac{\partial}{\partial t} Y_{^3\mathrm{He}}$$

4. **Express the abundance of ^{2}H in terms of that of ^{1}H and the relevant reaction rates λ_i.**
From Eq. 2, and $\frac{\partial}{\partial t} Y_{^2\mathrm{H}}$ we can solve for $Y_{^2\mathrm{H}}$:

$$Y_{^2\mathrm{H}} = \frac{Y_{^1\mathrm{H}}}{2} \frac{\lambda_{2\,^1\mathrm{H}\to{}^2\mathrm{H}}}{\lambda_{^1\mathrm{H}+{}^2\mathrm{H}\to{}^3\mathrm{He}}}$$

5. **Add specific values for the lambda's in order to determine the equilibrium value of $^2\mathrm{H}$**
Assume a central temperature of sun about $1.6{\times}10^7\,\mathrm{K}$, a central density about $160\,\mathrm{g\,cm^{-3}}$, and that the sun has burnt half of its $^1\mathrm{H}$ fuel by now.

Note: For a binary reaction we have $\lambda = N_\mathrm{A} \langle\sigma v\rangle \rho$
Obtain values for nuclear reaction rates from
`http://starlib.physics.unc.edu/RateLib.php`

Using
$\lambda_{2\,^1\mathrm{H}\to{}^2\mathrm{H}} = 1.044{\times}10^{-19} \times 160\,\mathrm{g\,mol^{-1}\,s^{-1}}$,
$\lambda_{^1\mathrm{H}+{}^2\mathrm{H}\to{}^3\mathrm{He}} = 1.482{\times}10^{-2} \times 160\,\mathrm{g\,mol^{-1}\,s^{-1}}$,
$Y_{^1\mathrm{H}} = 0.35$, we obtain

$$Y_{^2\mathrm{H}} = \frac{0.35\ \times\ 1.044{\times}10^{-19}}{2\ \times\ 1.482{\times}10^{-2}} = 1.233{\times}10^{-18}$$