Homework Set 2

Due: March 16, 2015, *before class*

Please do calculations and provide results using cgs units.

1. **The Eddington Limit**

   The Eddington limit is reached when, at the surface of the star, the acceleration due to the radiation pressure gradient balances gravitational acceleration.

   (a) **Derive the Eddington Luminosity based on this statement.**

   (b) For a mixture of hydrogen and helium, the electron scattering opacity is given by

   $$\kappa_{\mathrm{es}} = (1+X) \times 0.2\,\mathsf{cm}^2/g$$

   where X is the hydrogen mass fraction. For the sun X is about 70 % at the surface.
   Assuming the opacity at the surface of the sun where just due to electron scattering, compute the Eddington Luminosity of the sun.

   (c) **Based on last week's homework assignment, what is the Eddington luminosity of a human and how does it compare to the assumed "luminosity"?** Assume a hum has a hydrogen mass fraction of 10 %.

   (d) **If, very naively, one assumed a star did not deform due to rapid rotation, how is the Eddington limit (luminosity) modified, at the equator, for a rotating star?**

   (e) **What happens at the pole?**

   (f) **If a star cannot exceed the Eddington Luminosity corrected for centrifugal force at the equator (above question and assumptions), derive a maximum rotation rate for the star.**

2. **Neutrinos**

   Consider the following neutrino fluxes and energies from the sun:

Source		Flux at Earth ($\mathsf{cm}^2\,\mathsf{s}^{-1}$)	Energy (MeV)	Average (MeV)
$\mathsf{p}+\mathsf{p}$	$\longmapsto {}^2\mathsf{H}+\mathsf{e}^+ + \nu_\mathsf{e}$	6.0×10^{14}	≤ 0.42	0.263
${}^7\mathsf{Be}+\mathsf{e}^-$	$\longmapsto {}^7\mathsf{Li}+\nu_\mathsf{e}$	4.9×10^{13}	0.86 (90 %); 0.38 (10 %)	0.80
${}^8\mathsf{B}$	$\longmapsto {}^8\mathsf{Be}+\mathsf{e}^+ + \nu_\mathsf{e}$	5.7×10^{10}	≤ 15	7.2

   (a) **What is energy flux ($\mathsf{erg\,cm^{-2}\,s^{-1}}$) at the surface of the earth?**

   (b) **How many solar neutrinos are on average in a box of $1\,\mathsf{cm}^3$ on the surface of the earth at any given time?**
   Assume neutrinos move at the speed of light.

   (c) **What is the energy density from neutrinos (in $\mathsf{erg/cm^3}$) at the surface of the earth?**

3. **Nuclear Reaction Rates**
   Based on the general dependence of a non-resonant binary nuclear reaction,

   $$\langle\sigma v\rangle \propto (k_\mathsf{B}T)^{-2/3} \exp\left\{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m_{\mathrm{red}}}{k_\mathsf{B}T}\right)^{1/3}\right\}$$

   compute the temperature sensitivity of carbon burning, ^{12}C+^{12}C at $T = 10^9\,$K, that is, compute the exponent n in

   $$\langle\sigma v\rangle \propto T^n$$

   where n is given by

   $$n = \frac{\mathrm{d}\ln\langle\sigma v\rangle}{\mathrm{d}\ln T}\ .$$

   Here Z_i are the charges of the nuclei, e is elementary charge, h is the Planck constant, k_B the Boltzmann constant, and m_{red} the reduced mass of the two nuclei.

4. **Stellar Collapse.**

   Assume a star initially in hydrostatic equilibrium collapses to a black hole. For simplicity, let's assume each shell collapses to the center in the free-fall time scale (dynamical time scale) given by

$$\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\bar{\rho}(m)}}$$

   where $\bar{\rho}(m)$ is the average density inside mass coordinate m, and we neglect general relativity and pressure *during the collapse* (but not for the initial configuration of the star; "dust collapse"; Kippenhanh & Weigert, 1990, Eq. 27.10).

   Compute the mass accretion rate onto the central black hole as a function of mass coordinate, m, density $\rho(m)$, and average enclosed density, $\bar{\rho}(m)$.
   (At what accretion rate would the shell of the star at mass coordinate m accrete onto the central black hole?)

   Background: Consider the collapse of a massive star. When the very center collapses into a black hole, the rest of the star loses pressure support from below and collapses roughly on the free-fall time scale. In order to determine whether the star becomes a gamma-ray burst (GRB), we need to know the accretion rate onto the newly formed central black hole. Only if it is high enough will a gamma-ray burst result. Typically, the accretion rate needs to exceed some $0.1 - 1.0\,\mathsf{M}_{\odot}/\mathrm{yr}$. Knowing the accretion rate for each shell and the time for accretion of the shell, which is the free-fall time scale, we can now plot the accretion rate onto the black hole as a function of time, and estimate whether a gamma-ray burst should result or not.

   As another note, if you consider a star of constant density, the average density, and hence the free-fall time, would be identical for all shells, i.e., for the entire star. Hence, at first there would be no accretion, then all would accrete in one instance, then there would be no more accretion.

5. **Stellar Evolution Project**

   Get and install the MESA stellar evolution code
   `http://mesa.sourceforge.net/`

   The code uses `gfortran` (Linux, MacOS).

   Follow the installation instruction at `http://mesa.sourceforge.net/prereqs.html` including install of the SDK and compile the code.

   Try to run the examples at `http://mesa.sourceforge.net/starting.html`.