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# Example 1 Ions

$\mu$ # density

FAMILIAR : $P_I = n_I K_B T$

$K_B$ ← Boltzmann const.

Maxwell Boltzmann

$$n(p)\,dp = \frac{n_I \cdot 4\pi p^2 dp}{(2\pi m_I K_B T)^{3/2}} e^{-p^2/2m_I K T}$$

What is $n_I$?

$$n_I = \sum n_i = \sum_i \frac{\rho}{u} \cdot \frac{X_i}{A_i}$$

$X_i$ ← mass fraction of species i

$u$ ← atomic mass unit

$A_i$ ← mass of Ion / u

often we define $Y_i = \frac{X_i}{A_i}$ "mol/g"

DEF : MEAN MOLECULAR WEIGHT OF IONS

$$\frac{1}{\mu_I} = \sum_i \frac{X_i}{A_i} \longrightarrow n_I = \frac{\rho}{\mu_I u}$$

GAS CONSTANT $\mathcal{R} = \frac{K_B}{u} \left[ = K_B \cdot \frac{1g}{mol} \cdot N_A \right]$

$$P_I = \frac{\mathcal{R}}{\mu_I} \cdot \rho T$$

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## ELECTRONS

$$P_e = n_e k_B T$$

$$n_e = \sum z_i n_i = \frac{\rho}{u} \sum X_i \frac{z_i}{A_i} = \frac{\rho}{u} \sum z_i Y_i$$

DEF MEAN MOLECULAR WEIGHT PER ELECTRON

$$\frac{1}{\mu_e} = \sum_i z_i \frac{X_i}{A_i} = \sum_i z_i Y_i$$

$$\rightarrow n_e = \frac{\rho}{\mu_e \cdot u}$$

EXAMPLE: ASSUME WE HAVE H: $A = 1$, $z = 1$

REST $A \simeq 2 \cdot z$

$\rightarrow$ DERIVE FORMULA FOR $\mu_e$ (or $\frac{1}{\mu_e}$)

$$P_e = \frac{\mathcal{R}}{\mu_e} \cdot \rho T$$

RESULT: $\frac{1}{\mu_e}\ 1 + (1-X)\frac{1}{2} = \frac{1}{2}(1+X)$

TOTAL [GAS]

$$P_{GAS} = P_I + P_e = \mathcal{R} T \rho \underbrace{\left( \frac{1}{\mu_e} + \frac{1}{\mu_I} \right)}_{=: \frac{1}{\mu}} = \frac{\mathcal{R} T \rho}{\mu}$$

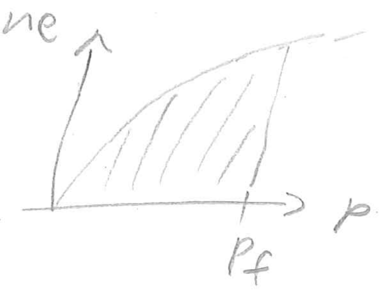
$$\rightarrow \frac{1}{\mu} = \frac{1}{\mu_e} + \frac{1}{\mu_I}$$

# DEGENERATE $e^-$



Heisenberg: $\Delta V \cdot \Delta p^3 \gtrsim h^3$

$$n_e(p) = \frac{2^{\leftarrow \text{spin}}}{h^3} \cdot 4\pi p^2 dp$$




## Fermi Momentum

$$P_f: \quad n_e = \int_0^{P_f} n_e(p)\, dp = \frac{8\pi}{3h^3} P_f^3$$

$$\Rightarrow P_f = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

Using: $v = p/m_e$:

$$P_{e,deg} = \frac{8\pi}{15 m_e h^3} P_f^5 = \underbrace{\frac{h^2}{20 m_e}\left(\frac{3}{\pi}\right)^{2/3} \frac{1}{u^{5/3}} \left(\frac{\rho}{\mu_e}\right)^{5/3}}_{=: K_1 \cdot \rho^{5/3}}$$

Rel Deg: $v \longrightarrow c \quad (E \gg m_e c^2)$

$$P = \frac{1}{3}\int_0^\infty n(p)\, c \cdot p \cdot dp = \underbrace{\frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} \frac{1}{u^{4/3}} \left(\frac{\rho}{\mu_e}\right)^{4/3}}_{=: K_2 \cdot \rho^{4/3}}$$

Q: When do we make the transition?
deg $\Leftrightarrow$ rel deg?
On what does it depend? $[T, \rho, \ldots]$?