20150306-1

Radiation

Planck: $n(\nu)d\nu = \frac{8\pi\nu^2}{c^3} \frac{d\nu}{e^{\frac{h\nu}{kT}}-1}$

↑ per vol, per $d\nu$

Pressure

$$P = \frac{1}{3}\int_0^\infty c\,\frac{h\nu}{c}\,n(\nu)\,d\nu = \frac{1}{3}aT^4$$

$$a = \frac{8\pi^5 k^4}{15c^3h^3} = \frac{4\sigma}{c}$$

Energy

(recall: $u = \frac{1}{\rho}\int_0^\infty n(p)\,\mathcal{E}(p)\,dp$

REL. Energy: $\mathcal{E} = mc^2\left[\left(1+\frac{p^2}{m^2c^2}\right)^{1/2}-1\right]$

limit $p \ll c \rightarrow p^2/2m$

or: $dp = \frac{h}{c}d\nu$

$$\rightarrow n(\nu)d\nu = n(p)\,dp = \frac{8\pi p^2}{h^3}\frac{dp}{e^{\frac{pc}{kT}}-1}$$

photons: $\mathcal{E} = h\nu$, $p = \frac{h\nu}{c}$

$$E = \frac{1}{\rho}\int_0^\infty h\nu\,n(\nu)\,d\nu = \frac{1}{\rho}aT^4$$

NON-REL GAS

[deg + non-deg] $$U_{GAS} = \frac{3}{2} \frac{P_{GAS}}{\rho}$$

We find interesting relations:

REL GAS: $$U_{GAS} = 3 \frac{P_{GAS}}{\rho}$$

RADIATION: $$U_{RAD} = \frac{aT^4}{\rho} = 3 \frac{P_{RAD}}{\rho}$$

→ NOTE REL GAS & RAD. GAS BEHAVE THE SAME!

Q: what happens @ very high T?

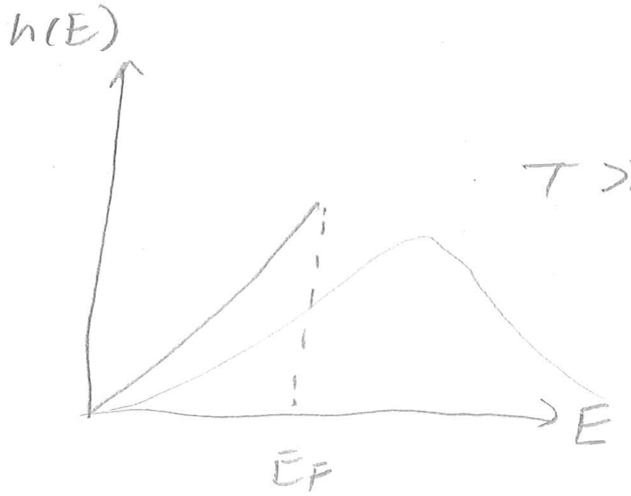
Q: Derive summary formula Ions + e^- [non-deg.]

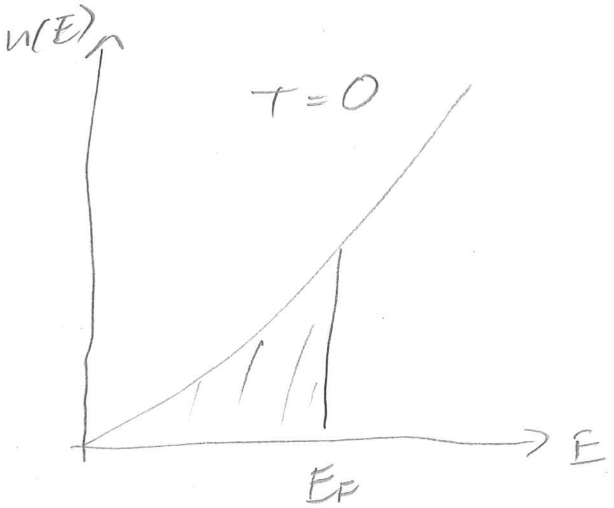
$$\mu^{-1} = \sum_i (Z_i + 1) Y_i$$

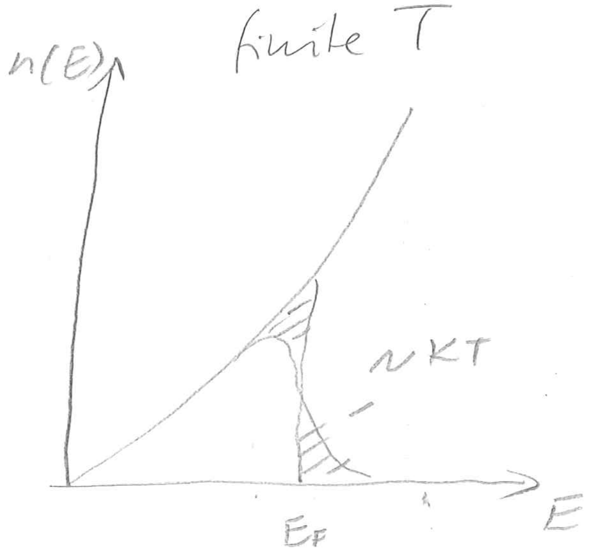
Q: Where are the different regimes important

EXAMPLE

DEG. GAS

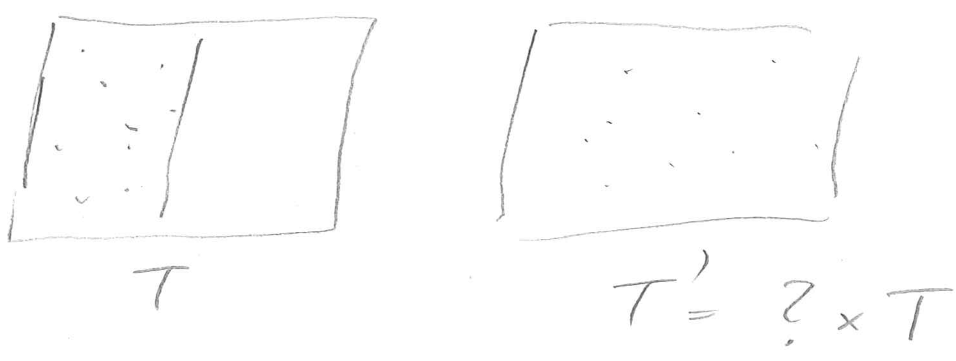






Q: For Ideal gas

DEGENERATE GAS



ADIABATIC EXPONENT

$du = T ds - P dv$

$v = 1/\rho$ specific volume
s : specific entropy

adiabatic : $\longleftrightarrow$ $ds = 0$ | Q: what does adiabatic mean?

$$0 = du + P d(1/\rho)$$

in "simple" systems, as discussed so far: $u \sim P/\rho$

$$\rightarrow u = \phi \cdot P/\rho$$

$$du = \phi \frac{1}{\rho} dP + \phi \cdot P\, d(1/\rho)$$

$$\phi \frac{1}{\rho} dP + \phi P d(\tfrac{1}{\rho}) + P d(\tfrac{1}{\rho}) = 0$$

$$\phi (1/\rho) dP + (1+\phi) P d(1/\rho) = 0 \qquad \Big| \; d\frac{1}{\rho} = -\frac{1}{\rho^2} d\rho$$

$$\phi (\tfrac{1}{\rho}) dP = (\phi + 1) \frac{1}{\rho^2} P \, d\rho$$

$$\frac{\phi + 1}{\phi} = \frac{\rho \, dP}{P \, d\rho} = \left.\frac{d \ln P}{d \ln \rho}\right)_{ad} = \gamma_{ad}$$

Q: Compute adiabatic INDEX FOR

- Ideal gas
- RAD. gas
- DEG. gas
- REL. DEG. gas

| Q: Why do we care?

OTHER EXPONENTS

$$\frac{\Gamma_2}{\Gamma_2 - 1} : \left.\frac{d\ln P}{d\ln T}\right)_{ad} = \frac{1}{\nabla_{ad}}$$

$$\Gamma_3 = \left.\frac{d\ln T}{d\ln \rho}\right)_{ad} + 1$$

Q: are these all independent?

Why?

$$\frac{\Gamma_1}{\Gamma_3 - 1} = \frac{\Gamma_2}{\Gamma_2 - 1}$$

CRYSTALLIZATION

Compare Coulomb energy to thermal E

thermal $k_B T$

Coulomb $\frac{q^2}{d}$ $q = Z e$

d? $\frac{4\pi r^3}{3}$: specific volume

Volume per particle: $\frac{g}{m} = \frac{g}{A \cdot u} = \frac{1}{}\frac{4\pi}{3} r^3 \simeq 4 r^3$

$\Gamma = \frac{Z^2 e^2}{d\, K T}$ Coulomb Parameter

$\Gamma \sim$ few (1...5) gas $\rightarrow$ liquid

$\sim$ 180 ... 185 liquid $\rightarrow$ solid

cf: Pair correlation function