We had:

$$F = \underbrace{-\frac{4ac}{3}\frac{T^3}{\kappa\rho}}_{=: K_{RAD}}\frac{\partial T}{\partial r}$$

K_{RAD} RADIATIVE CONDUCTIVITY

IN ASTRO:

USE temperature gradient $\frac{\partial T}{\partial P}$

$$\frac{\partial T/\partial m}{\partial P/\partial m} = \frac{3}{16\pi acG}\frac{\kappa l}{mT^3}$$

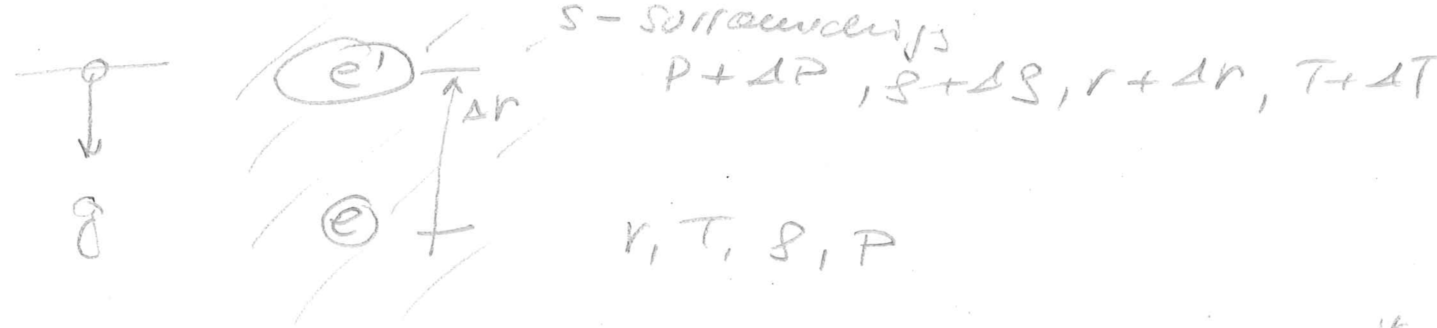
DEF: $\nabla_{RAD} = \left.\frac{\partial \ln T}{\partial \ln P}\right)_{RAD} = \frac{3}{16\pi acG}\frac{\kappa l P}{mT^4}$

$$\rightarrow \frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla_{RAD})$$

Later we will see that this formalism can also be used to describe energy transport in other regions

Q: other modes of energy transport?

Local Dynamic (In) Stability



STABILITY ANALYSIS PROCEDURE: no heat exchange *
Display "EDDIE" ADIABATICALLY, then ask:
Is it buoyant?

* Dynamic stability analysis

CONDITION: $\boxed{\rho'_e > \rho_s}$

ASSUME:
- PRESURE EQUILIBRIUM
- NO composition exchange

Let's look @ small changes:

$\left|\frac{d\rho}{dr}\right|_e < \left|\frac{d\rho}{dr}\right|_s$ | $\left(\frac{d\rho}{dr}\right)_e > \left(\frac{d\rho}{dr}\right)_s$ | NOTE $\frac{d\rho}{dr} < 0$

FOR STABILITY!

Q: What do we do next?

GENERAL EOS   ASSUME IDEAL GAS WITH RADIATION   20150311 – 3

$$\frac{d\varrho}{\varrho} = \alpha \cdot \frac{dP}{P} - \delta \frac{dT}{T} + \varphi \cdot \frac{d\mu}{\mu}$$

with $\alpha := \left.\frac{\partial \ln \varrho}{\partial \ln P}\right|_{T,\mu}$; $\delta := -\left.\frac{\partial \ln \varrho}{\partial \ln T}\right|_{P,\mu}$; $\varphi := \left.\frac{\partial \ln \varrho}{\partial \ln \mu}\right|_{P,T}$

Q: what do these quantities mean?

Put in:

$$\left.\frac{\alpha}{P}\frac{dP}{dr}\right|_e - \left.\frac{\delta}{T}\frac{dT}{dr}\right|_e + \left.\frac{\varphi}{\mu}\frac{d\mu}{dr}\right|_e > \left.\frac{\alpha}{P}\frac{dP}{dr}\right|_s - \left.\frac{\delta}{T}\frac{dT}{dr}\right|_s + \left.\frac{\varphi}{\mu}\frac{d\mu}{dr}\right|_s$$

0

Pressure equilibrium: P-terms cancel

Def Pressure scale height:

hydrostatic equil

$$H_P := -\frac{dr}{d\ln P} = -P\frac{dr}{dP} = -\frac{P}{g\varrho} > 0$$

multiply by H_P:

$$\delta \left.\frac{d\ln T}{\partial \ln P}\right|_e > \delta \left.\frac{d\ln T}{d\ln P}\right|_s - \varphi \cdot \left.\frac{d\ln \mu}{d\ln P}\right|_s$$

DEF: $\nabla := \left.\frac{d\ln T}{d\ln P}\right|_s$; $\nabla_e := \left.\frac{d\ln T}{d\ln P}\right|_e$; $\nabla_\mu := \left.\frac{d\ln \mu}{d\ln P}\right|_s$

symbol, not operator

$$\rightarrow \quad \nabla < \nabla_e + \frac{\varphi}{\delta}\nabla_\mu$$

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Specifically $\cdot\ \nabla = \nabla_{RAD}$ FOR stable structure

$\cdot\ \nabla_e = \nabla_{ad}$ adiabatic displacement

→ good approximation in convective region

$$\boxed{\nabla_{RAD} < \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu}$$

LEDOUX CRITERION FOR STABILITY against convection

w/o composition gradients $\boxed{\nabla_\mu = 0}$

$$\boxed{\nabla_{RAD} < \nabla_{ad}}$$

SCHWARZSCHILD CRITERION for stability

Reasoning: If you mix stuff, comp. gradients are wiped out

→ in that sense, Ledoux is not "reversible" — once mixed, Schwarzschild applies!

Semiconvection

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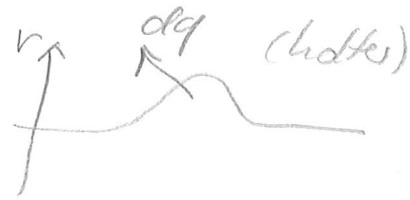
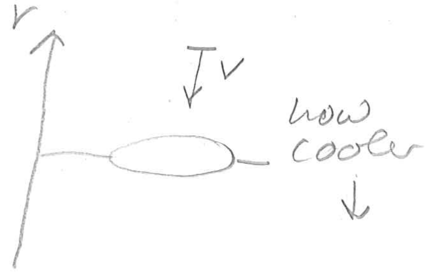
Stabilised by composition gradient

→ oscillatory instability

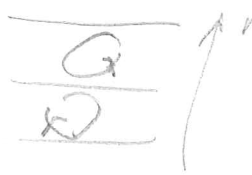




eventually: enough energy in oscillation for full turnover

→ Layer formation

→ effective transport of heat but composition trapped in layers



(mean free path of photons >> ions)

$$\tau_{Diff} = \ell^2/D$$

· on long time scale: merging of layers