Homework Set 2

Due: March 14, 2016, *before class*

Please do calculations and provide results using cgs units.

1. **Crystallization**

   Consider a gas of $^{56}\mathsf{Fe}$ at a temperature of $10^8\,\mathsf{K}$.

   (a) **Estimate at what density will it transition from gas to liquid ($\Gamma = 1$) and at what temperature it will transition from liquid to crystal ($\Gamma = 180$).**

   (b) **For each density, estimate the Fermi Energy of electron gas. Is the electron gas degenerate? Is the iron Ion gas degenerate. Please argue your case.**

   (c) **Would you expect electron captures on the iron to occur at either of these densities? Why?**

2. **Equation of State.**

   Consider a photon gas with energy density $U = aT^4$ and pressure $P = \frac{a}{3}T^4$.

   Using the first and second laws of thermodynamics, $\mathrm{d}u = T\mathrm{d}s - P\mathrm{d}v$, derive the adiabatic index for such a photon gas.

   NOTE: Here u is the specific energy, $v = 1/\rho$ is the specific density, s is the specific entropy, a is the radiation constant, and T is the temperature.

3. **Neutrinos**

   Consider the following neutrino fluxes and energies from the sun:

Source		Flux at Earth $(\mathsf{m}^{-2}\,\mathsf{s}^{-1})$	Energy (MeV)	Average (MeV)
$\mathsf{p}+\mathsf{p}$	$\longmapsto {}^2\mathsf{H}+\mathsf{e}^+ + \nu_\mathsf{e}$	6.0×10^{14}	≤ 0.42	0.263
$^7\mathsf{Be}+\mathsf{e}^-$	$\longmapsto {}^7\mathsf{Li}+\nu_\mathsf{e}$	4.9×10^{13}	0.86 (90 %); 0.38 (10 %)	0.80
$^8\mathsf{B}$	$\longmapsto {}^8\mathsf{Be}+\mathsf{e}^+ + \nu_\mathsf{e}$	5.7×10^{10}	≤ 15	7.2

   (a) **What is energy flux ($\mathsf{erg\,cm^{-2}\,s^{-1}}$) at the surface of the earth?**

   (b) **How many solar neutrinos are on average in a box of $1\,\mathsf{cm}^3$ on the surface of the earth at any given time?**
   Assume neutrinos move at the speed of light.

   (c) **What is the energy density from neutrinos (in $\mathsf{erg/cm^3}$) at the surface of the earth?**

4. **Nuclear Reaction Rates**
   Based on the general dependence of a non-resonant binary nuclear reaction,

$$\langle\sigma v\rangle \propto (k_\mathsf{B}T)^{-2/3}\exp\left\{-\frac{3}{2}\left(\frac{4\pi^2 Z_1 Z_2 e^2}{h}\right)^{2/3}\left(\frac{m_\mathrm{red}}{k_\mathsf{B}T}\right)^{1/3}\right\}$$

   compute the temperature sensitivity of carbon burning, ^{12}C+^{16}O at $T = 2\times10^9$ K, that is, compute the exponent n in

$$\langle\sigma v\rangle \propto T^n$$

   where n is given by

$$n = \frac{\mathrm{d}\ln\langle\sigma v\rangle}{\mathrm{d}\ln T}\,.$$

   Here Z_i are the charges of the nuclei, e is elementary charge, h is the Planck constant, k_B the Boltzmann constant, and m_red the reduced mass of the two nuclei.

5. **Stellar Collapse.**

Assume a star initially in hydrostatic equilibrium collapses to a black hole. For simplicity, let's assume each shell collapses to the center in the free-fall time scale (dynamical time scale) given by

$$\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\bar{\rho}(m)}}$$

where $\bar{\rho}(m)$ is the average density inside mass coordinate m, and we neglect general relativity and pressure *during the collapse* (but not for the initial configuration of the star; "dust collapse"; Kippenhanh & Weigert, 1990, Eq. 27.10).

Compute the mass accretion rate onto the central black hole as a function of mass coordinate, m, free fall time, τ_{ff}, density $\rho(m)$, and average enclosed density, $\bar{\rho}(m)$.
(At what accretion rate would the shell of the star at mass coordinate m accrete onto the central black hole?)

6. **Stellar Evolution Project**

Get and install the MESA stellar evolution code
`http://mesa.sourceforge.net/`

The code uses `gfortran` (Linux, MacOS).

Follow the installation instruction at `http://mesa.sourceforge.net/prereqs.html` including install of the SDK and compile the code.

Try to run the examples at `http://mesa.sourceforge.net/starting.html`.