Homework Set 3

Due: March 21, 2016, *before class*

Please do calculations and provide results using cgs units.

1. **Simple Stellar Models**

   Write a program to solve the Lane-Emden Equation.

   Suggested steps are:

   (a) Convert Lane-Emden Equation into a system of 1st order linear differential equations.

   (b) find inner boundary condition as needed for integration; formally the term is 0/0, so you have to find the right limit by Taylor expansion or other method.

$$\lim_{\xi \to 0} \frac{\mathrm{d}^2\theta(\xi)}{\mathrm{d}\xi^2} = \ldots$$

   (c) Write integration program using finite steps. Don't just use pre-compiled solver package. It is reasonably possible to do this just with a spreadsheet like Google Documents (which is what I did for a proof of concept). Obviously, you are most welcome to use Python or another common programming language (FORTRAN, C, C++) instead.

   (d) Find value of ξ where θ crosses 0. In the lecture we called this value ξ_1. If you use finite steps you could use interpolation to refine the estimate. Also obtain

$$\left.\frac{\mathrm{d}\theta(\xi)}{\mathrm{d}\xi}\right|_{\xi_1} .$$

   Show and document you solution steps and program (submit program or formulas used in spreadsheet).

   Tasks:

   (a) Compute a model for $n = 1.5$, $M = 1\,\mathrm{M}_\odot$, central density $\rho_\mathrm{c} = 160\,\mathrm{g\,cm^{-3}}$. Use ideal gas with $\mu = 1.3$. **What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star?**

   (b) Compute a model for $n = 1.5$, $M = 10\,\mathrm{M}_\odot$, $\rho_\mathrm{c} = 1.6\,\mathrm{g\,cm^{-3}}$, and a mixture of 70 % hydrogen (^{1}H), 28 % helium (^{4}He) and 2 % of nitrogen (^{14}N), all percentages by mass fraction. Use ideal gas *with radiation.* Assume the gas is fully ionized.

   What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star? What is fraction of gas to total pressure,

$$\beta = \frac{P_\mathrm{gas}}{P}$$

   at the center of the star?

   (c) **Compare your solar model from above with key data of the actual sun. What might be reasons for the discrepancy?**

2. **Mass Excess and Binding Energy.**

   A table of mass excess is given at
   `http://2sn.org/Class/ASP4000-Stars-2015-S1/homework/mass.mas03`
   The table lists in each line Z, A, and mass excess per nucleus in keV.

   (a) **Which is the most tightly bound nucleus (largest binding energy per nucleon) and which which is the most stable nucleus (most negative mass excess *per nucleon*)?**

   (b) **Why do the differences in highest binding energy and lowest mass excess arise, and what is their nuclear physics origin and implication?**

   (c) **In the table Li3 has negative BE - what does this mean?**

3. **Stellar Evolution Project**

   Get and install the MESA stellar evolution code
   `http://mesa.sourceforge.net/`

   The code uses `gfortran` (Linux, MacOS).

   Follow the installation instruction at `http://mesa.sourceforge.net/prereqs.html` including install of the SDK and compile the code.

   Try to run the examples at `http://mesa.sourceforge.net/starting.html`.

   Submit some example outputs from your test runs.