Homework Set 4

Due: April 6, 2016, *before class*

Please do calculations and provide results using cgs units.

1. **Stellar Model with Thermonuclear Burning**

   Based on the Lane Emden Equation solver developed for Homework Set 3, construct stellar models that include nuclear burning.

   Assume a composition that is a mixture of 70 % hydrogen (^{1}H), 28 % helium (^{4}He) and 2 % of nitrogen (^{14}N), by mass fraction.

   (a) Compute a model with $n = 1.5$ and $M = 1\,\mathrm{M}_{\odot}$. Assume the star generates its energy according to the pp chains. Use formula 18.63 from Kippenhahn, Weigert, & Weiss (2012). For simplicity, assume $\psi = 1$ and $f_{11} = 2$. Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

   Find central density such that the luminosity of star equal the solar luminosity. What is the central temperature of the star? Plot specific nuclear energy generation rate and luminosity as a function of mass coordinate. What is the radius of the star? Why does it deviate from that of the sun?

   (b) Compute a model with $n = 3$ and $M = 1000\,\mathrm{M}_{\odot}$. Assume the star generates its energy according to the CNO cycle. Use formula 18.65 from Kippenhahn, Weigert, & Weiss (2012). Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

   What is the luminosity and radius of the star for a central temperature of $2\times10^{7}\,\mathrm{K}$, $2.5\times10^{7}\,\mathrm{K}$, $3\times10^{7}\,\mathrm{K}$, and $3.5\times10^{7}\,\mathrm{K}$?

2. **Nuclear Reaction Kinematics - The Hot CNO Cycle**
   Based on the general formula for nuclear reactions,

$$\frac{\partial}{\partial t}Y_i = \sum_{\substack{\alpha_1,\alpha_2,\ldots\\ \beta_1,\beta_2,\ldots}} \lambda_{\alpha_1\mathbf{1}+\alpha_2\mathbf{2}+\ldots\to\beta_1\mathbf{1}+\beta_2\mathbf{2}+\ldots}\frac{\beta_i-\alpha_i}{\alpha_1!\alpha_2!\ldots}Y_1^{\alpha_1}Y_2^{\alpha_2}\ldots$$

   (a) **Write down the system of equations for the changes of all species in the β-limited (hot) CNO cycle assuming steady state. Assume a net production of one (1) ^{4}He nucleus from four (4) ^{1}H nuclei.**

   (b) **Assuming steady state (also for the rest of this problem), what are the changes of each of the cycle nuclei ($Z \geq 6$) with respect to time?**

   (c) Find the half life of two radioactive nuclei in the cycle. As a characteristic of the hot cycle, these radioactive decays are by far the slowest processes in the cycle. Assume an initial composition of 70 % ^{1}H, 28 % ^{4}He, 2 % ^{14}N by *mass fraction*. **How long does it take to burn all ^{1}H to ^{4}He?** Derive and show first an analytic expression, then add values and compute the numerical result.

   (d) **What is an astrophysical interpretation of the time scale derived above?**

   (e) **Using the assumptions from above, after all the hydrogen has burnt and all radioactive decays have completed, what is the mass fraction of each of the relevant species?**