Homework Set 5

Due: April 13, 2016, *before class*

Please do calculations and provide results using cgs units.

1. **Simple Model for Helium Burning Reactions**
   Based on the general formula for nuclear reactions,

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \to \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

   (a) **Write the equations for the rate of change of helium, carbon, and oxygen due to the triple alpha and the $^{12}\mathsf{C}(\alpha,\gamma)^{16}\mathsf{O}$ reactions.**

   (b) Let us assume a simple one-zone model for helium burning. At the beginning helium burning, assume an initial helium abundance of 100 %. **Assuming the reaction rates ($\lambda_{\ldots}$) for the 3α and $^{12}\mathsf{C}(\alpha,\gamma)^{16}\mathsf{O}$ reactions are identical (in cgs units; e.g., set them to 1), compute and plot the abundances of $^4\mathsf{He}$, $^{12}\mathsf{C}$, and $^{16}\mathsf{O}$ as a function of time from the beginning of helium burning to the end of helium burning using a logarithmic time axis. Adapt reasonable truncations, including for the final $^4\mathsf{He}$ abundance, e.g., a mole fraction of 1×10^{-6}. Make another plot showing $^{12}\mathsf{C}$, and $^{16}\mathsf{O}$ abundances as a function of $^4\mathsf{He}$ abundance.**

   (c) **Using formula 18.67 and 18.70 from KWW12 and assuming $T = 2\times10^8\,\mathsf{K}$, $\rho = 1\times10^3\,\mathsf{g\,cm^{-3}}$, compute the temperature sensitivity of the two reactions rates ($\lambda \sim T^n$, $n = \ldots$). Use the energy release per reaction as provided to convert the energy generation rates provided into reaction rates, λ. Start by giving the expressions for λ for each case.** Assume $f_{12,4} = f_{3\alpha} = 1$.

   (d) **Using these reaction rates, make the same plots as above in 1b. To verify your results, please check mass conservation!**

   (e) Bonus: **Compute and plot the final carbon-to-oxygen *mass fraction ratio* on a grid of temperatures and densities relevant to hydrostatic burning in massive stars.** How do you interpret the result?

2. Helium Burning in Real Stellar Models

   (a) **Using the MESA code, follow the Evolution of a $15\,\mathsf{M}_\odot$ and $25\,\mathsf{M}_\odot$ star of initial solar composition to the end of central helium burning (at least central $^4\mathsf{He}$ mass fraction has dropped to 1×10^{-4} or, better, central temperature has reached $5\times10^8\,\mathsf{K}$). Plot central temperature and central density *during central helium burning* as a function of time.** Feel free to provide other plots you like, e.g., if you can make a Kippenhahn diagram (KD). If any of you is interested to convert the MESA output into an input format for my KD plots, that would be welcome as well and you can get the python module for my plotting routines from me.

   (b) **Make the same plots as in 1b for the *central* abundances of $^4\mathsf{He}$, $^{12}\mathsf{C}$, and $^{16}\mathsf{O}$, but using the stellar evolution data of the above simulations.**

   (c) **How do the results compare to each other and to the analytic model? What are the key differences in input to the reactions and how does this affect the outcome? Please provide arguments or calculations to support your conclusions.**