Lecture I: EOS

Reading: (KWW 2012)
§§ 4.1–4.3, 8.1, 13, 15, 16
optional § 14 (ionization)

Goal: · Being able to build stellar model
· Compute stellar evolution
→ need to know relevant physics
→ need to know simple analytic solutions so we can check model (verification, validation...)

· build simple stellar model (Lane Emden)
· use stellar evolution code to run own models

Q: What would you need to make a model of a star?
→ where do we start?

Equation of State (EOS)

- What is EOS?

→ composition + independent variables
(e.g. T, ρ — intensive quantities
→ give P, E, … | Q: How many & which do we need?

- What [constituents] do we have to consider
  - Gas: ideal, degenerate
    Q: what gases (Ions, electrons, … composition ?!
  - Radiation

But: Change of Composition due to
- ionisation
- nuclear reactions
  → strong, weak forces, [What does each do?

chemical or nuclear equilibrium
↙
Q: How are these important for EOS
Do they change EOS?
→ can be incorporated in EOS (Q: when, how?
Q: What does this mean?! How to do?

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→ EOS has many components...
How to combine?

Example: Total pressure

Simple case: $P_{TOT} = \underbrace{\sum_i P_i}_{\text{Partial Pressures}}$

Q: is this always so?

Q: What part do/can/should we study separately?

How do we best divide things up?

Typical case:

$$P = P_{RAD} + \underbrace{P_{IONS} + P_{e^-}}_{P_{GAS}}$$

Q: Do you know terms for certain conditions?

Similar for E:

$$E = E_{RAD} + \underbrace{E_{IONS} + E_{e^-}}_{E_{gas}}$$

But let's derive in detail...

PRESSURE

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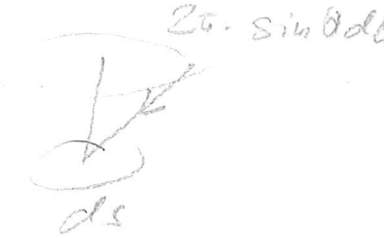

1) fraction of particles hitting ds at angle θ:

$$\frac{2\pi \sin\theta \, d\theta}{4\pi} = \frac{1}{2} \sin\theta \, d\theta$$

4π ← all possible directions

2) change in momentum:

$$dp = 2 \cdot \cos\theta \cdot p = 2 p_\perp$$

3) rate at which particles hit the surface

rate = $\cos\theta \cdot v(p)$

$$\rightarrow P = \underbrace{\int_0^{\pi/2} 2 \cdot \cos\theta \cdot \frac{1}{2} \sin\theta \cos\theta \, d\theta}_{1/3} \times \ldots \times \int_0^\infty n(p) \, v(p) \cdot p \, dp$$

$$n(p): \frac{\#\text{ Particles}}{\text{Volume} \times dp}$$

dp ← $(p, p+dp)$ interval

Energy density

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Similar ... but E is not vector

→ no geometric factor from projection

$$u = \frac{1}{\rho}\int_0^\infty n(p)\, \mathcal{E}(p)\, dp$$

specific internal energy

energy of particle with momentum p

Q: why do we integrate over p not v?
(consider specific case, gases, ...)