Example 1 Ions

FAMILIAR: $P_I = n_I \cdot k_B \cdot T$

(n_I ← # density; k_B ← Boltzmann const.)

Maxwell Boltzmann

$$n(p)\,dp = \frac{n_I \cdot 4\pi p^2 dp}{(2\pi m_I k_B T)^{3/2}} e^{-p^2/2m_I kT}$$

What is n_I?

$$n_I = \sum n_i = \sum_i \frac{\rho}{u} \frac{X_i}{A_i}$$

(X_i ← mass fraction; A_i ← mass of ion/u; u ← atomic mass unit)

Often we define: $Y_i = \frac{X_i}{A_i}$ "mol/g"

DEF: mean molecular weight of Ions:

$$\frac{1}{\mu_I} = \sum_i \frac{X_i}{A_i} \rightarrow n_I = \frac{\rho}{\mu_I u}$$

Gas Constant $R = \frac{k_B}{u}$

$$P_I = \frac{R}{\mu_I} \cdot \rho \cdot T$$

ELECTRONS

$$P_e = n_e \cdot k_B T$$

$$n_e = \sum_i Z_i n_i = \frac{\rho}{u} \sum_i X_i \frac{Z_i}{A_i} = \frac{\rho}{u} \sum_i Z_i Y_i$$

DEF MEAN MOLECULAR WEIGHT PER ELECTRON

$$\frac{1}{\mu_e} \equiv \sum_i Z_i \frac{X_i}{A_i} = \sum_i Z_i Y_i$$

$$\rightarrow n_e = \frac{\rho}{\mu_e \cdot u}$$

EXAMPLE ASSUME WE HAVE H: $A=1,\ Z=1$

REST $A \simeq 2 \cdot Z$

$\rightarrow$ derive FORMULA FOR μ_e (OR $\frac{1}{\mu_e}$)

RESULT: $\frac{1}{\mu_e} = X + (1-X) \cdot \frac{1}{2} = \frac{1}{2}(1+X)$

$$P_e = \frac{\mathcal{R}}{\mu_e} \cdot \rho \cdot T$$

TOTAL [GAS]

$$P_{GAS} = P_I + P_e = \mathcal{R} T \rho \underbrace{\left(\frac{1}{\mu_e} + \frac{1}{\mu_I}\right)}_{= \frac{1}{\mu}} = \frac{\mathcal{R} T \rho}{\mu}$$

$$\rightarrow \boxed{\frac{1}{\mu} = \frac{1}{\mu_e} + \frac{1}{\mu_I}}$$

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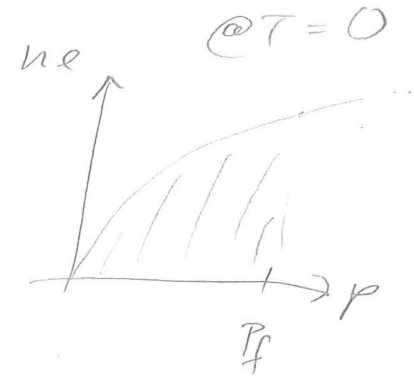
DEGENERATE e^-



HEISENBERG : $\Delta V \cdot \Delta^3 p \gtrsim h^3$

(ΔV ← volume)

$$n_e(p)\,dp = \frac{2}{h^3} \cdot 4\pi p^2\,dp$$

(2 ← spin)

FERMI MOMENTUM

$$P_f : n_e = \int_0^{P_f} n_e(p)\,dp = \frac{8\pi}{3h^3} P_f^3 \doteq \frac{\rho}{\mu_e \cdot u}$$

$$\rightarrow P_f = \left(\frac{3h^3 n_e}{8\pi}\right)^{1/3}$$

USING $v = P/m_e$

$$P_{e,deg} = \frac{8\pi}{15 m_e h^3} P_f^5 = \underbrace{\frac{h^2}{20 m_e}\left(\frac{3}{\pi}\right)^{2/3} \frac{1}{u^{5/3}}}\left(\frac{\rho}{\mu_e}\right)^{5/3}$$

$$=: K_1 \cdot \rho^{5/3}$$

REL DEG: $v \longrightarrow c$ $\quad (E >> m_e c^2)$ $\quad v = c \cdot \sqrt{\frac{p^2}{m^2c^2+p^2}}$

$$P = \frac{1}{3}\int_0^\infty n(p) \cdot c \cdot p \cdot dp = \underbrace{\frac{hc}{8}\left(\frac{3}{\pi}\right)^{1/3} \frac{1}{u^{4/3}}}\left(\frac{\rho}{\mu_e}\right)^{1/3}$$

$$=: K_2 \cdot \rho^{4/3}$$

Q: When do we make the transition?

deg ⟷ rel. deg ?

On what does it depend? [T, ρ, ...] ?