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per vol, per $d\nu$

PLANCK: $n(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3}\frac{d\nu}{e^{\frac{h\nu}{kT}}-1}$

PRESSURE: $P = \frac{1}{3}\int_0^\infty c\cdot\frac{h\nu}{c}\,n(\nu)\,d\nu = \frac{1}{3}aT^4$

with $a = \frac{8\pi^5 k_B^4}{15c^3h^3} = \frac{4\sigma}{c}$

ENERGY DENSITY (recall: $u = \frac{1}{\rho}\int_0^\infty n(p)\,\varepsilon(p)\,dp$

REL. ENERGY: $\varepsilon = mc^2\left[\left(1+\frac{p^2}{m^2c^2}\right)^{1/2}-1\right]$

limit $p \ll c$: $\rightarrow p^2/2m$

OR: $dp = \frac{h}{c}\,d\nu$

$\rightarrow n(\nu)\,d\nu = n(p)\,dp = \frac{8\pi p^2}{h^3}\frac{dp}{e^{\frac{pc}{kT}}-1}$

photon: $\varepsilon = h\nu$, $p = \frac{h\nu}{c}$

$E = \frac{1}{\rho}\int_0^\infty h\nu\cdot n(\nu)\,d\nu = \frac{1}{\rho}\,a\cdot T^4$

non-REL GAS

(degenerate + non-degenerate)

$$U_{GAS} = \frac{3}{2} \frac{P_{GAS}}{\rho}$$

REL GAS:

$$U_{GAS} = 3 \cdot \frac{P_{GAS}}{\rho}$$

We find interesting relations!

Radiation:

$$U_{RAD} = \frac{aT^4}{\rho} = 3 \frac{P_{RAD}}{\rho}$$

→ NOTE REL GAS & RAD GAS BEHAVE THE SAME!

Q: What happens @ very high T?

Q: Derive summary formula Ions + e^-
[non-deg]

$$\mu^{-1} = \sum_i (Z_i + 1) X_i$$

Q: Where are the different regimes important?

Q: Do we deal with deg nuclei?
if so – where, or why usually not
(physics...)

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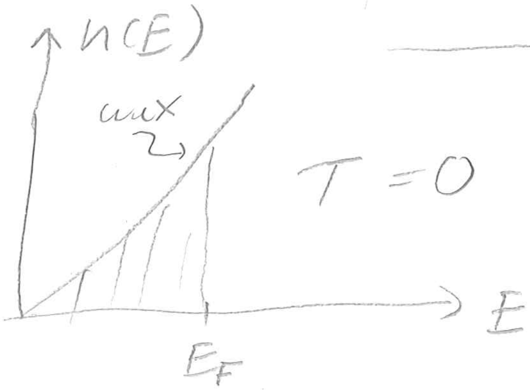
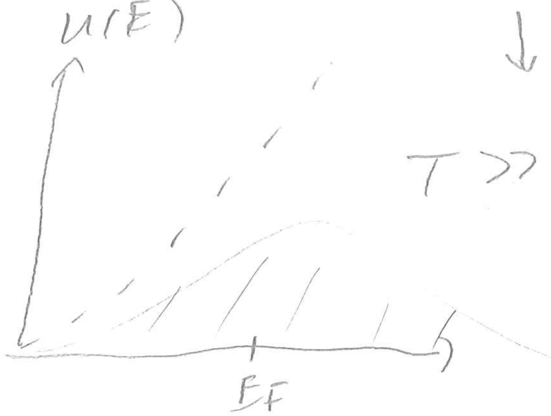
EXAMPLE

$\boxed{\text{DEG GAS}}$

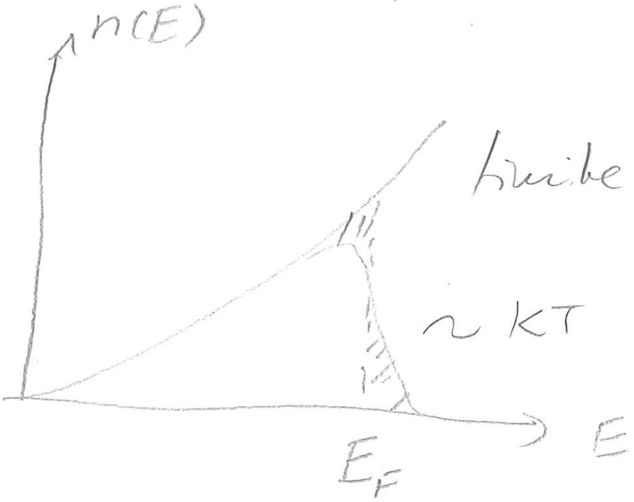


$T = 0$

derive



$T \gg \frac{E_F}{k_B}$



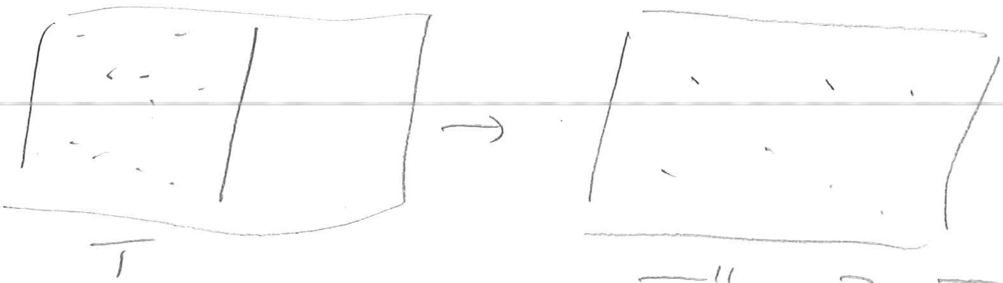
finite $0 < T \lesssim \frac{E_F}{k_B}$

Q: For Ideal gas:

remove wall

$T' = ? \times T$

DEGENERATE GAS



$T'' = ? \cdot T$

Adiabatic Exponent

$du = T ds - P dv$ | $v = 1/\rho$: specific volume

adiabatic: $\leftrightarrow$ $ds = 0$ | s: specific entropy

| Q: What does "adiabatic" mean?

$$0 = du + P d(1/\rho)$$

in "simple" systems, as discussed so far: $u \sim P/\rho$

$$\rightarrow u = \phi \cdot P/\rho$$

$$du = \phi \frac{1}{\rho} dP + \phi \cdot P d(1/\rho)$$

$$0 = \phi \frac{1}{\rho} dP + \phi P d(1/\rho) + P d(1/\rho)$$

$$0 = \phi \left(\frac{1}{\rho}\right) dP + (1+\phi) P d(1/\rho) \quad \Big| \; d\frac{1}{\rho} = -\frac{1}{\rho^2} d\rho$$

$$\phi \left(\frac{1}{\rho}\right) = (1+\phi) \frac{1}{\rho^2} P d\rho$$

$$\frac{\phi+1}{\phi} = \frac{\rho dP}{P d\rho} = \frac{d \ln P}{d \ln \rho}\Big)_{ad} =: \gamma_{ad}$$

Q: Compute adiabatic INDEX FOR

- Ideal Gas
- RAD gas
- DEG gas
- REL DEG GAS

| Q: Why do we care?

Other Exponents.

$$\frac{\Gamma_2}{\Gamma_2 - 1} = \left.\frac{d\ln P}{d\ln T}\right)_{ad} = \frac{1}{\nabla_{ad}} \quad \text{adiabatic } T \text{ gradient}$$

$$\Gamma_3 = \left.\frac{d\ln T}{d\ln \rho}\right)_{ad} + 1$$

Q: are these all independent? why?

$$\frac{\Gamma_1}{\Gamma_3 - 1} = \frac{\Gamma_2}{\Gamma_2 - 1}$$

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CRYSTALLIZATION

Compare Coulomb energy to thermal E

thermal: $k_B T$

Coulomb: $\frac{q^2}{d}$ $\quad q = Ze$

d? $\frac{4\pi r^3}{3}$ — specific volume

volume per particle: $\frac{\rho}{m} = \frac{\rho}{Au} \stackrel{!}{=} \frac{4\pi}{3} \sim 4r^3$

$$\Gamma = \frac{Z^2 e^2}{d\, k_B T}$$ Coulomb Parameter

$\Gamma \sim$ few (1...5) gas $\leftrightarrow$ liquid

$\sim 180 \ldots 185$ liquid $\leftrightarrow$ solid

cf. Pair correlation function

Q: what happens in mixtures

(consider alloys!)