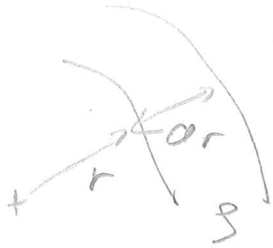
STELLAR STRUCTURE

1) MASS COORDINATE

$V_{sphere} = \frac{4\pi}{3} R^3$



→ implicit mass conservation e.g., in codes

$\rightarrow dV = 4\pi r^2 dr$

$dm = \rho \cdot dV = 4\pi r^2 \rho dr$

$$\boxed{\frac{dm}{dr} = 4\pi r^2 \rho}$$

$\boxed{m: \text{Lagrange COORDINATE}}$

use: $\frac{d}{dm} = \frac{1}{4\pi r^2 \rho} \frac{d}{dr}$ for COORDINATE TRANSFORMS

NB

Sometimes people also use ... used to use ...

$q = \frac{m}{M}$ ← total mass

but: not Lagrangian!

→ mass conservation is more difficult to formulate

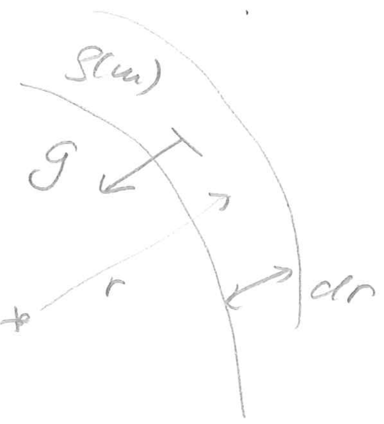
e.g. mass loss accretion

Assume structure & stellar properties are function of t and m

→ Coupled PDE system

2) Momentum Conservation

[hydrostatic equilibrium]



$$g = -\frac{Gm(r)}{r^2} = \dots \frac{Gm}{r(m)^2}$$

$$-\frac{\partial P}{\partial r} + g\rho = 0 \leftrightarrow \text{no net acceleration}$$

$$\frac{\partial P}{\partial r} = g\rho = -\frac{Gm(r)}{r^2} \cdot \rho \quad \Big| \quad \frac{\partial}{\partial m} = \frac{1}{4\pi r^2 \rho}\frac{\partial}{\partial r}$$

$$\boxed{\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}}$$

Q: Estimate P_c, T_c in terms of average ρ: $\bar{\rho}$
USE IDEAL GAS and central ρ: ρ_c
(plus M, R)
... mean values...

$P_0 = 0 \qquad m \sim M/2$

$P_c = ? \qquad r \sim R/2$

$$P_c = \frac{16\,GM}{2 \cdot 4\pi R^4} = \frac{2GM^2}{\pi R^4}$$

$$\bar{\rho} = \frac{3M}{4\pi R^3}$$

$$P_c = \frac{\mathcal{R}\, T_c\, \rho_c}{\mu}$$

$$= \frac{8G\bar{\rho}M}{3R}$$

$$T_c = \frac{P_c}{\rho_c}\frac{\mu}{\mathcal{R}} = P_c \frac{\mu}{\mathcal{R}} \frac{\bar{\rho}}{\rho_c} \cdot \frac{4\pi R^3}{3M}$$

typical $\frac{\bar{\rho}}{\rho_c} \ll 1$

$$\simeq \frac{8}{3}\frac{\mu}{\mathcal{R}}\frac{GM}{R}\frac{\bar{\rho}}{\rho_c}$$

Hydrodynamic Version

20160307-3

$$\frac{\partial P}{\partial r} = g\rho - \rho\frac{\partial^2 r}{\partial t^2}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

W/O PRESSURE: FREE FALL.

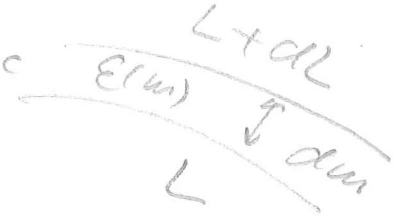
$$\tau_{FF} \simeq \left(\frac{R}{g}\right)^{1/2} \simeq \frac{1}{2}\frac{1}{\sqrt{G\bar{\rho}}} \quad \text{Q. son..}$$

Spherical collapse:

$$\tau_{FF} = \sqrt{\frac{3\pi}{32G\bar{\rho}(m)}}$$

Energy Conservation

$$dL = 4\pi r^2 \rho \, \varepsilon \, dr = \varepsilon \, dm$$ ← specific



$$\frac{\partial L}{\partial m} = \varepsilon = \varepsilon_{nuc} + \varepsilon_\nu + \varepsilon_g$$

where "gravothermal" energy is given by

$$\varepsilon_g = -T\frac{\partial s}{\partial t} = -c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}$$

where we used:

$$dq = c_P \, dT - \frac{\delta}{\rho} dP$$

specific volume $V = \frac{1}{\rho}$

$$\alpha = \left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T \; , \quad \delta = \left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P$$

Q: what do these quantities α and δ actually mean? How do we interpret them

ESTIMATE OF STELLAR ENERGY 20160307-5

Kelvin Helmholtz time-scale

$$\tau_{KH} = \frac{E_G}{L} \overset{(*)}{\simeq} \frac{2E_i}{L}$$ ← internal energy

$$|E_G| \simeq \frac{G\bar{m}}{\bar{r}} \simeq \frac{GM^2}{2R} \quad ; \quad \bar{m} \simeq M/2, \quad \bar{r} \simeq R/2$$

$$\rightarrow \tau_{KH} \simeq \frac{GM^2}{2RL}$$ Sun $\sim 1.6 \times 10^7$ yr

(*) VIRIAL THEOREM : HW read up on it!

Energy transport 20160307-6

Concept: mean free path of photon

$$l_{ph} = \frac{1}{\rho \kappa}$$ typical for sun: 2cm

Diffusive flux: $j = -D \nabla n$

- $D = \frac{1}{3} v \cdot l_{ph}$
- ∇n ? use energy density $U = aT^4$

$$\rightarrow \frac{\partial u}{\partial r} = 4aT^3 \frac{\partial T}{\partial r}$$

$\rightarrow$ diffusive flux $j \rightarrow F = -\frac{4ac}{3}\frac{T^3}{\kappa\rho}\frac{\partial T}{\partial r}$

$=: K_{RAD}$

use: $l(m) = 4\pi r^2(m) F$ luminosity

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi a c r^2}\frac{\kappa \rho l}{T^3} \quad | r \rightarrow m$$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 a c}\frac{\kappa l}{r^4 T^3}$$