We had

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$$F = -\underbrace{\frac{4ac}{3}\frac{T^3}{\kappa\rho}}_{K_{RAD}}\frac{\partial T}{\partial r}$$

Radiative conductivity

IN ASTRO:

use temperature gradient $\frac{\partial T}{\partial P}$

$$\frac{\partial T/\partial m}{\partial P/\partial m} = \frac{3}{16\pi acG} \times \frac{\kappa l}{m T^3}$$

DEF $\nabla_{RAD} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_{RAD} = \frac{3}{16\pi acG}\frac{\kappa l P}{m T^4}$

$$\rightarrow \frac{\partial T}{\partial m} = -\frac{Gm}{4\pi P}\nabla_{RAD}$$

NOTE: Later we will see that this formalism can also be used to describe energy transport in other regimes

Q: what other modes of energy transport may there be?

Local Dynamic (IN) STABILITY

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S – surroundings
$P+\Delta P,\ \rho+\Delta\rho,\ r+\Delta r,\ T+\Delta T$

r, T, ρ, P

Stability analysis procedure:

Displace "eddy" adiabatically, then ask

no heat (*) exchange

"IS IT BUOYANT?"

(*): Dynamic stability analysis

condition: $\boxed{\rho'_e > \rho_s}$

ASSUME:
- Pressure equilibrium
- No composition exchange

let's look @ small changes:

$$\left.\frac{d\rho}{dr}\right|_e > \left.\frac{d\rho}{dr}\right|_s$$

NOTE:

$$\frac{d\rho}{dr} < 0$$

FOR STABILITY

Q: what do we do next?

Generical EOS 20160308 - 3

(for now: assume ideal gas with radiation)

$$\frac{d\varrho}{\varrho} = \alpha \cdot \frac{dP}{P} - \delta \cdot \frac{dT}{T} + \varphi \cdot \frac{d\mu}{\mu}$$

with $\alpha := \left.\frac{d\ln\varrho}{d\ln P}\right|_{T,\mu}$; $\delta := -\left.\frac{d\ln\varrho}{d\ln T}\right|_{P,\mu}$; $\varphi := \left.\frac{d\ln\varrho}{d\ln\mu}\right|_{P,T}$

Q: what do these quantities mean?

PUT IN:

$$\frac{\alpha}{P}\left.\frac{dP}{dr}\right|_e - \frac{\delta}{T}\left.\frac{dT}{dr}\right|_e + \frac{\varphi}{\mu}\frac{d\mu}{dr_e} > \frac{\alpha}{P}\left.\frac{dP}{dr}\right|_s - \frac{\delta}{T}\left.\frac{dT}{dr}\right|_s + \frac{\varphi}{\mu}\left.\frac{d\mu}{dr}\right|_s$$

Pressure equilibrium: P-terms cancel

Def: Pressure scale height:

$$H_P := \frac{dr}{d\ln P} = -P\frac{dr}{dP} \overset{\text{hydrostatic equilibrium}}{=} -\frac{P}{g\varrho} > 0$$

multiply by H_P:

$$\delta\left.\frac{d\ln T}{d\ln P}\right|_e > \delta\left.\frac{d\ln T}{\partial\ln P}\right|_s - \varphi\left.\frac{d\ln\mu}{d\ln P}\right|_s$$

DEF: $\nabla := \left.\frac{d\ln T}{d\ln P}\right|_s$; $\nabla_e := \left.\frac{d\ln T}{d\ln P}\right|_e$

symbol, not operator ; $\nabla_\mu := \left.\frac{d\ln\mu}{d\ln P}\right|_s$

$$\rightarrow \quad \nabla < \nabla_e + \frac{\varphi}{\delta}\nabla_\mu$$

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Specifically: • $\nabla = \nabla_{rad}$ for stable structure

• $\nabla_e = \nabla_{ad}$ adiabatic displacement

→ good approximation in convective regions

$$\boxed{\nabla_{RAD} < \nabla_{ad} + \frac{\varphi}{\delta} \nabla_\mu}$$

LEDOUX criterion for STABILITY against convection

w/o composition gradients: $\boxed{\nabla_\mu = 0}$

$$\boxed{\nabla_{RAD} < \nabla_{ad}}$$

SCHWARZSCHILD CRITERION for stability

Reasoning for Schwarzschild:

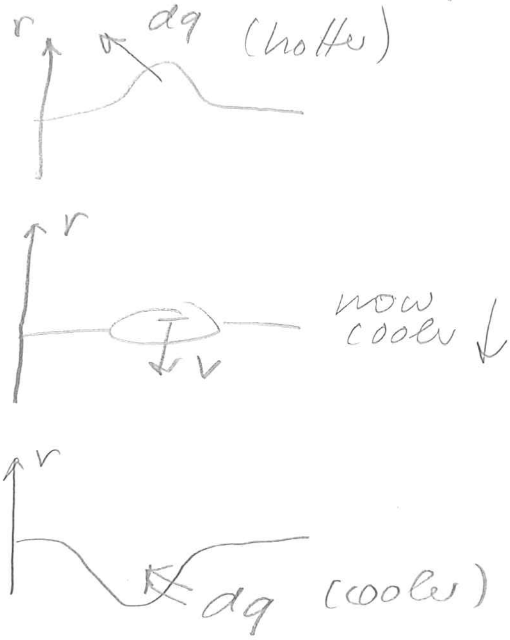
If you mix stuff, composition gradients are wiped out

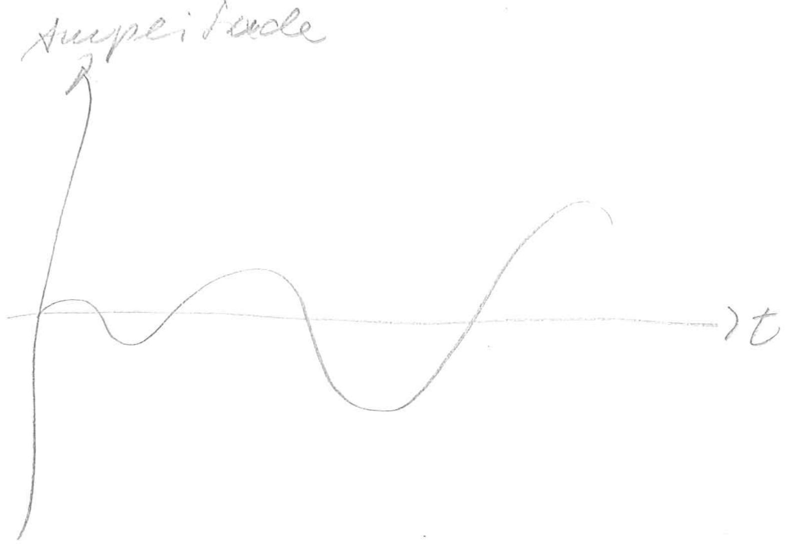
→ in that sense, Ledoux is not "reversible": once mixed, SCHWARZSCHILD APPLIES

Semiconvection

stabilised by composition gradient

→ oscillatory instability





eventually: enough energy in oscillation for full turn-over

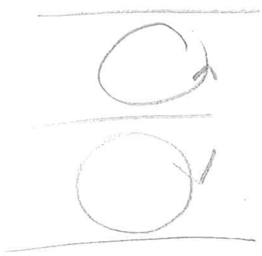
→ layer formation

→ effective transport of heat

but composition trapped in layers

(mean free path of photons >> ions)

$\tau_{diff} = \ell^2/D$



• on long time scale: merging of layers