NOTES ON NUCLEAR REACTIONS

$$\frac{\partial}{\partial t} Y_i = \sum_{\text{reactions}} \lambda_{\underbrace{\underbrace{\alpha_1 \mathbb{1} + \alpha_2 \mathbb{2} + \ldots}_{\text{IN}} \rightarrow \underbrace{\beta_1 \mathbb{1} + \beta_2 \mathbb{2} + \ldots}_{\text{OUT}}}_{\text{channel}}} \times \frac{\beta_i - \alpha_i}{\Pi_j \alpha_j} \Pi_j Y_j^{\alpha_j}$$

INDEX OF RATE

EXAMPLE

$$^{12}C + {}^{12}C \longrightarrow {}^{23}Na + p$$

$$\lambda_{2 \cdot {}^{12}C \rightarrow 1 \cdot {}^{23}Na + 1 \cdot p}$$

α_1 β_2 β_3

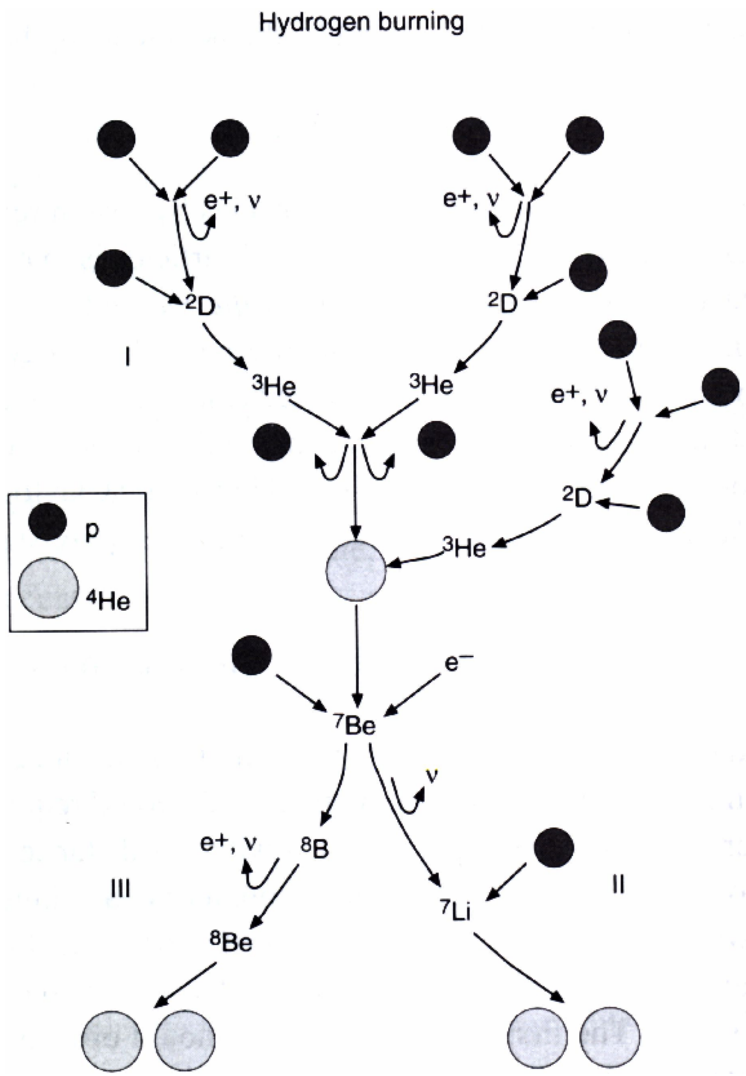
$$\frac{\partial}{\partial t} Y_{^{12}C} = \lambda_{2\,^{12}C \rightarrow {}^{23}Na + p} \frac{0-2}{2!} Y_{^{12}C}^2$$

$$= -\lambda_{2\,^{12}C \rightarrow {}^{23}Na + p} Y_{^{12}C}^2$$

$$\frac{\partial}{\partial t} Y_{^{23}Na} = \lambda_{2 \cdot {}^{12}C \rightarrow {}^{23}Na + p} \cdot \frac{1}{2} Y_{^{12}C}^2 = \frac{\partial}{\partial t} Y_p$$

if you have several reactions that change a species, <u>all</u> contributions have to be added!

Hydrogen-Burning: pp Chains



$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$$
$$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

$$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H} \quad (pp1)$$

$$^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma \tag{18.62}$$

$$^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu$$
$$^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} \quad (pp2)$$

$$^7\mathrm{Be} + {}^1\mathrm{H} \rightarrow {}^8\mathrm{B} + \gamma$$
$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + e^+ + \nu$$
$$^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He} \quad (pp3)$$

Energy release:

$Q(pp1) = 26.20 MeV$

$Q(pp2) = 25.67 MeV$

$Q(pp3) = 19.20 MeV$

Reaction rate: $\langle \sigma v \rangle \propto T^4$

Hydrogen Burning: CNO Bi-Cycle

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu \\
{}^{13}\mathrm{C} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu \\
{}^{15}\mathrm{N} + {}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He} \\
& {}^{16}\mathrm{O} + \gamma \\
{}^{16}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + e^{+} + \nu \\
{}^{17}\mathrm{O} + {}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}
\end{aligned}
$$

Energy release:
$Q(CNO) = 24.97 MeV$

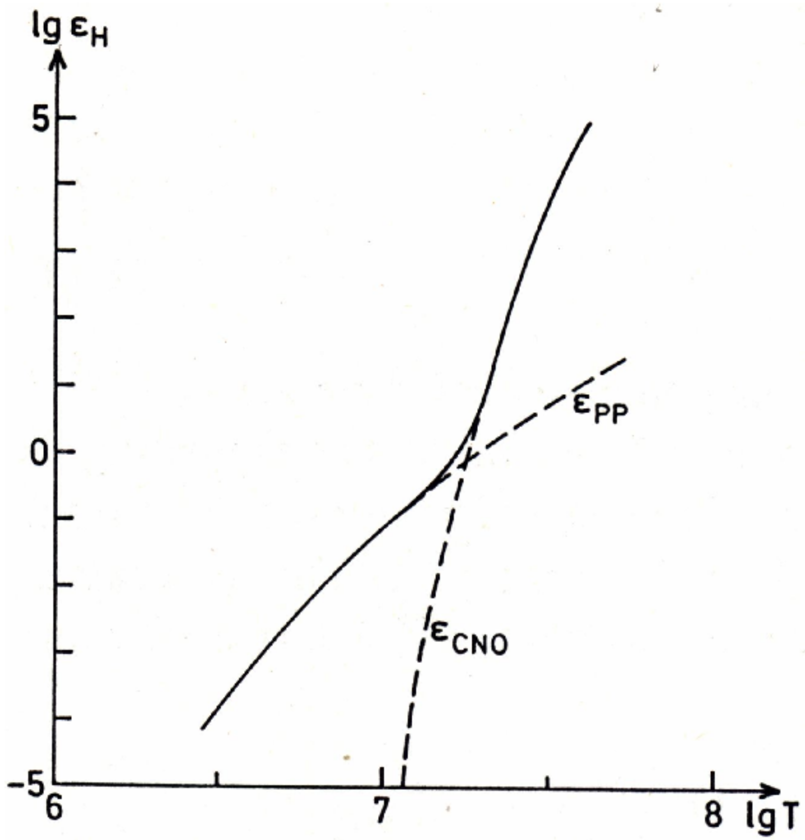
Reaction rate: $\langle \sigma v \rangle \propto T^{16}$

Branching:
CNO-1 : CNO-2 $\sim$ 10,000 : 1

Hydrogen Burning: CNO Bi-Cycle

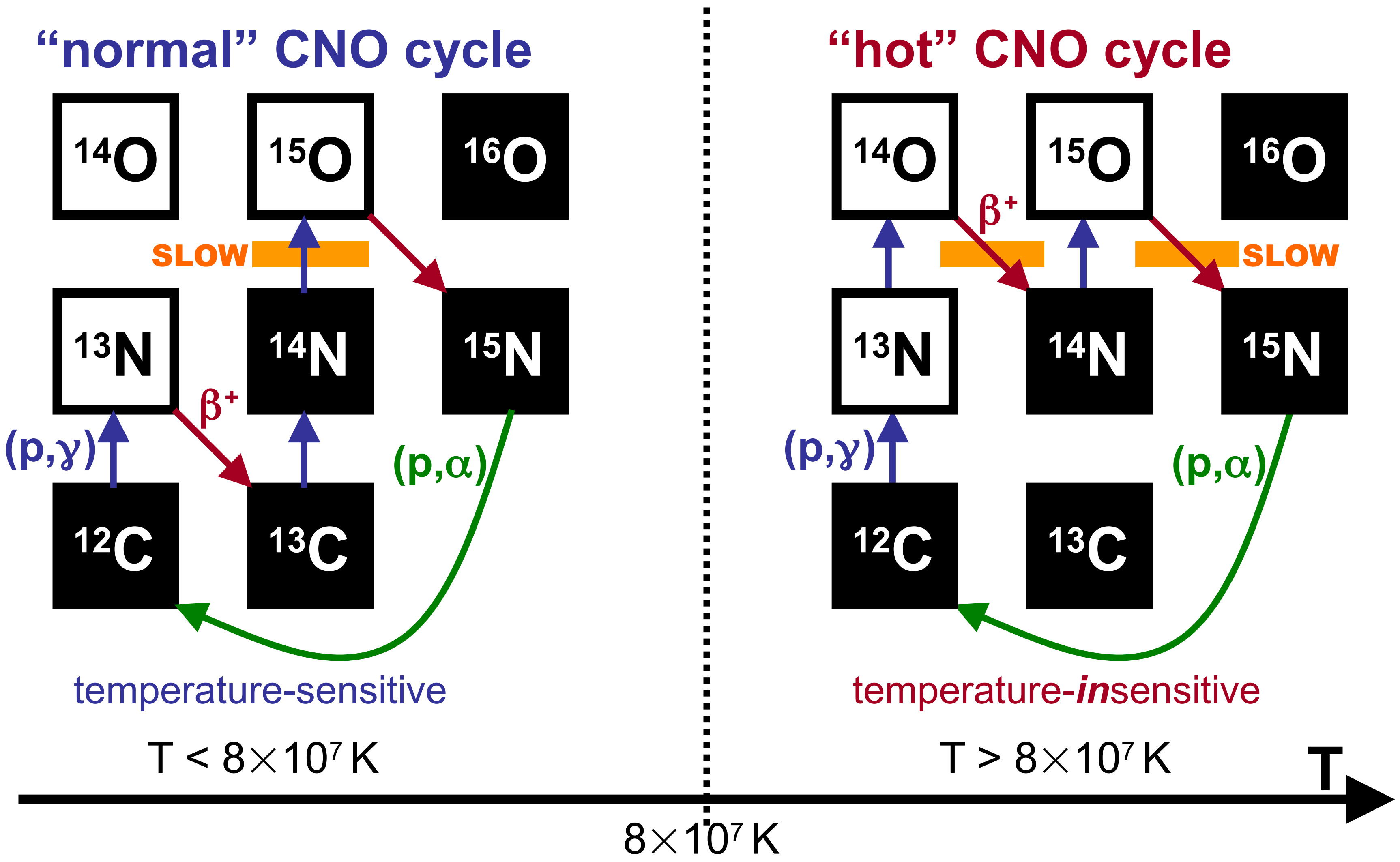
- Usually the beta-decays are fast compared to the capture reactions, (p,γ).
- ^{14}O: $\tau_{1/2} = 70$ sec
  ^{15}O: $\tau_{1/2} = 122$ sec
  ^{13}N: $\tau_{1/2} = 10\,$min
  ^{17}F: $\tau_{1/2} = 64$ sec
  ^{18}O: $\tau_{1/2} = 110\,$min
- ^{14}N(p,γ)^{15}O usually is the slowest "bottleneck" reaction.
- CNO cycle burning converts most CNO isotopes into ^{14}N.

Competition of Hydrogen-Burning Modes



Transition from pp-chains
in low-mass stars (low T)
to CNO chains
in high-mass stars (high T)

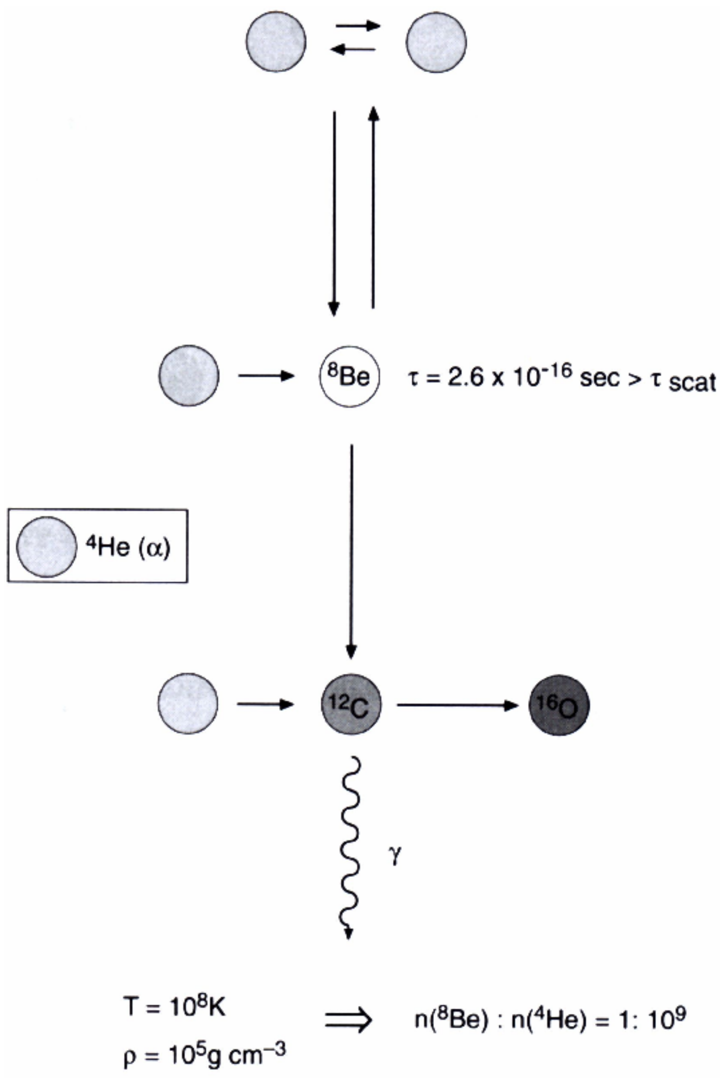
Hydrogen Burning by CNO Cycle



time for an eddy to burn its hydrogen content by **hot** CNO cycle

$$\tau_H = 11\,\mathrm{h}\left(\frac{0.02}{Z}\right)\left(\frac{X_0}{0.7}\right)$$

Helium Burning



Step 1:

$^4\text{He} + {}^4\text{He} \rightleftharpoons {}^8\text{Be}$

Built up equilibrium abundance of ^{8}Be

Lifetime of ^{8}Be is only 2.6×10^{-16} s!

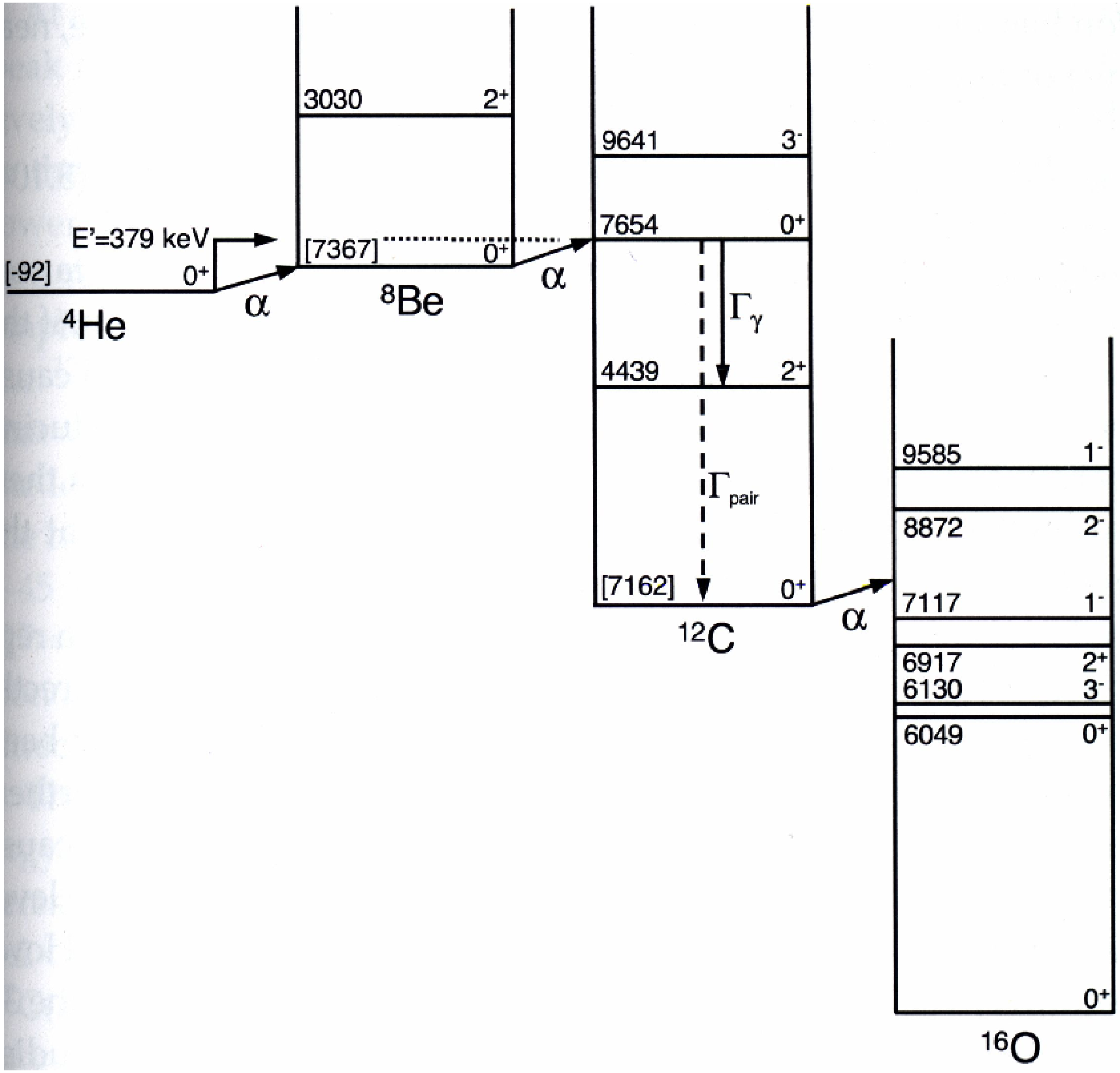
Step 2:

$^8\text{Be} + {}^4\text{He} \to {}^{12}\text{C} + \gamma$

$Q_{3\alpha} = 7.275\,\text{MeV}$

$< \sigma v > \propto \rho^2 T^{40}$

Helium Burning Level Scheme



Additional Helium Burning Reactions

Oxygen Production

$^{4}\mathrm{He} + {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} + \gamma$

$Q = 7.162\,\mathrm{MeV}$

$\langle \sigma v \rangle \ \propto \rho T^{40}$

The final abundance of carbon is set by the competition of 3α and $^{12}\mathrm{C}(\alpha, \gamma)^{16}\mathrm{O}$ reactions;

The production of $^{16}\mathrm{O}$ can only start when a sufficient amount of $^{12}\mathrm{C}$ has been made.