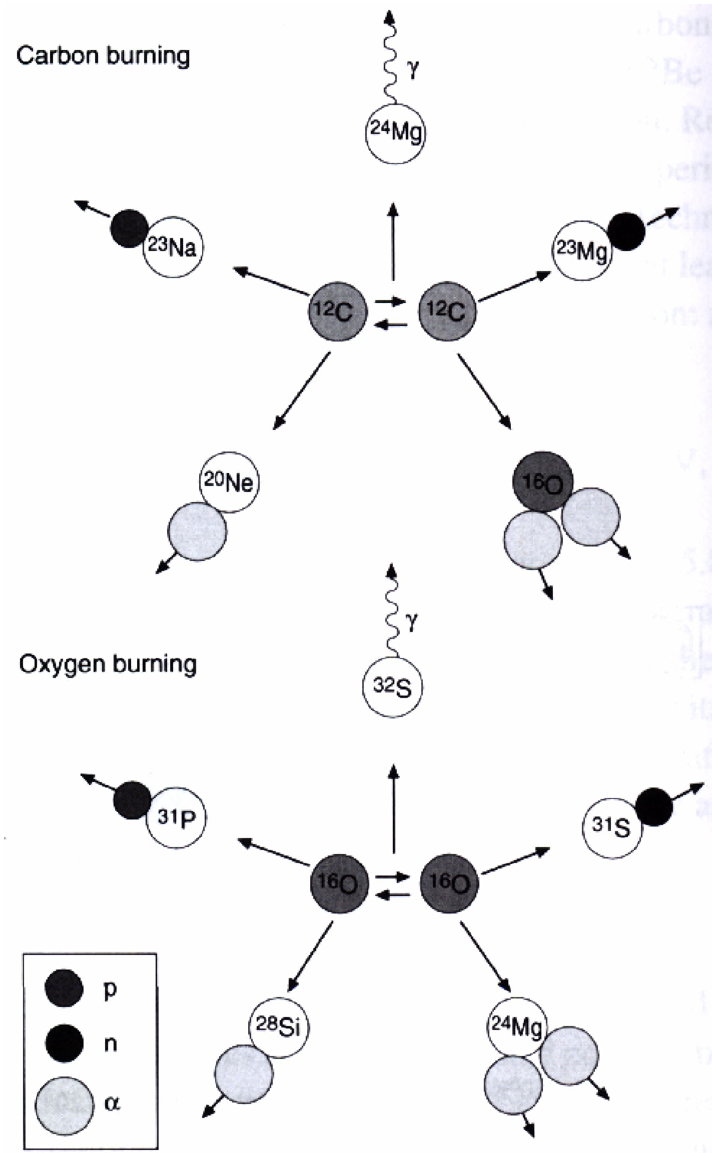
Carbon and Oxygen Burning



Carbon Burning

$$
\begin{aligned}
{}^{12}\mathrm{C} + {}^{12}\mathrm{C} &\rightarrow {}^{24}\mathrm{Mg} + \gamma \quad , & 13.931 \\
&\rightarrow {}^{23}\mathrm{Mg} + n \quad , & -2.605 \\
&\rightarrow {}^{23}\mathrm{Na} + p \quad , & 2.238 \\
&\rightarrow {}^{20}\mathrm{Ne} + \alpha \quad , & 4.616 \\
&\rightarrow {}^{16}\mathrm{O} + 2\alpha \quad , & -0.114
\end{aligned}
$$

Average $Q = 13\,\mathrm{MeV}$

Oxygen Burning

$$
\begin{aligned}
{}^{16}\mathrm{O} + {}^{16}\mathrm{O} &\rightarrow {}^{32}\mathrm{S} + \gamma \quad , & 16.541 \\
&\rightarrow {}^{31}\mathrm{P} + p \quad , & 7.677 \\
&\rightarrow {}^{31}\mathrm{S} + n \quad , & 1.453 \\
&\rightarrow {}^{28}\mathrm{Si} + \alpha \quad , & 9.593 \\
&\rightarrow {}^{24}\mathrm{Mg} + 2\alpha \quad , & -0.393
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$$

Average $Q = 16\,\mathrm{MeV}$

Neutrino losses from electron/positron pair annihilation

- Important for carbon burning and beyond

- For $T > 10^9$ K (about 100 keV), occasionally:

  $\gamma \rightarrow e^+ + e^-$

  and usually

  $e^+ + e^- \rightarrow 2\gamma$

  but sometimes

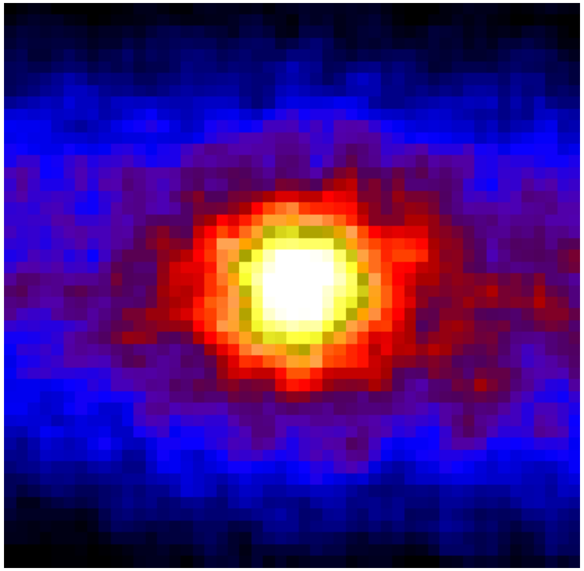
  $e^+ + e^- \rightarrow \overline{\nu}_e + \nu_e$

-
- The neutrinos exit the stars at the speed of light while the e^+, e^-, and the γ's all stay trapped.

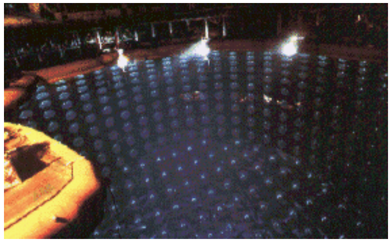
- This is an important energy loss with

  $\varepsilon_\nu \approx -10^{15}\ (T/10^9 K)^9$ erg g^{-1} s^{-1}

- For carbon burning and beyond, each burning stage gives about the same energy per nucleon, thus the lifetime goes down as T^{-9}



The sun as seen by Kamiokande



Neon Burning

Neon burning proceeds by a combination of photo-disintegrations and α captures:

$$^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + {}^{4}\text{He}\,, \quad Q = -4.73\,\text{MeV}$$

This reaction dominates over the inverse reaction known from helium burning for $T > 1.5 \times 10^9\,\text{K}$.

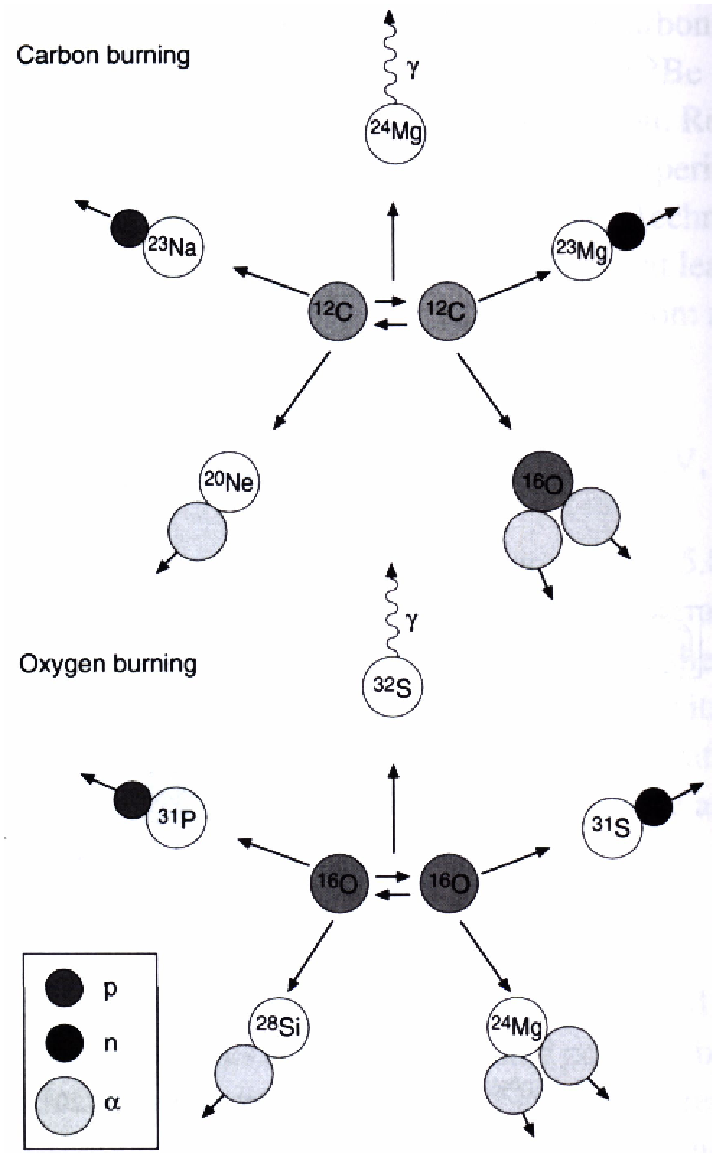
Subsequently, the ^{4}He is captured on another ^{20}Ne nucleus:

$$^{20}\text{Ne} + {}^{4}\text{He} \rightarrow {}^{24}\text{Mg} + \gamma.$$

The net result is

$$2\,{}^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + {}^{24}\text{Mg} + \gamma\,, \quad Q = +4.583\,\text{MeV}$$

Carbon and Oxygen Burning



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Silicon/Sulfur Burning

Actually, often we have more sulfur in the star than there is silicon, but it is custom to call this phase "silicon burning".

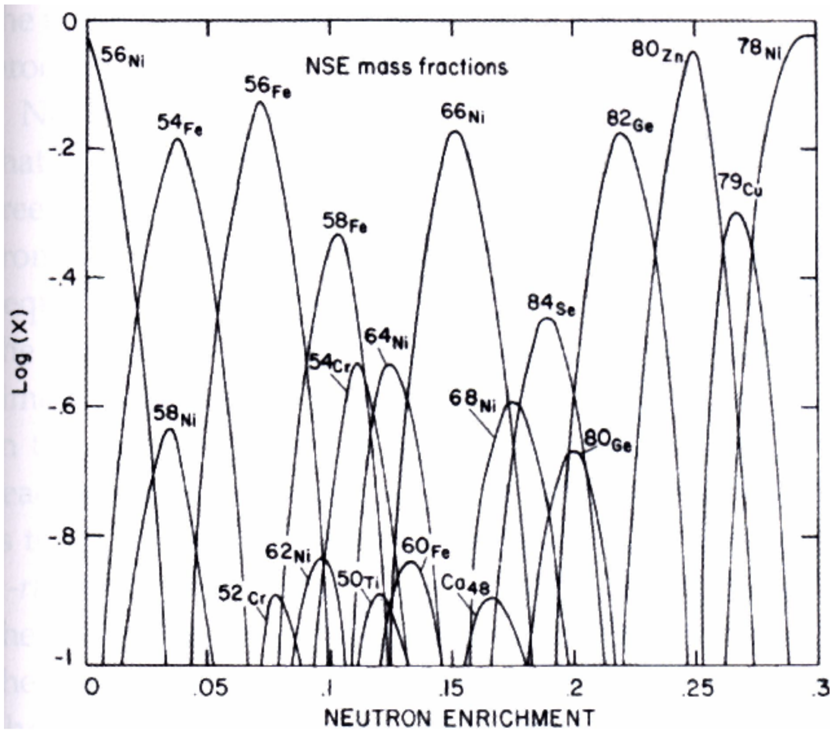
Typical burning temperature is $3 \ldots 3.5 \times 10^9$ K.

Similar to neon burning, silicon burning proceeds as a series of photo-disintegration reactions, mostly, (γ, α), and helium capture reactions, (α, γ) to build up iron group elements.

$$(\gamma, \alpha) \rightleftharpoons (\alpha, \gamma)$$

At the high T and ρ of these conditions, also *weak reactions* occur, converting protons into neutrons and leading to a *neutron excess*. This allows to actually make stable iron isotopes.

Beyond Silicon Burning



NSE distribution for
$T = 3.5 \times 10^9$ K,
$\rho = 10^7$ g/cm^3

After silicon burning T and ρ is so high that the nuclei are in **nuclear statistical equilibrium**, i.e., the reactions are fast compared to the evolution time-scale of the star, and the abundance distribution of the nuclei is given by a *Saha equation*.

Summary of Energies

Nuclear Fuel	*Process*	$T_{threshold}$ 10^6 K	*Products*	*Energy per Nucleon (MeV)*
H	*p–p*	~4	He	6.55
H	CNO	15	He	6.25
He	3α	100	C, O	0.61
C	C + C	600	O, Ne, Na, Mg	0.54
O	O + O	1000	Mg, S, P, Si	~0.3
Si	Nuc. eq.	3000	Co, Fe, Ni	<0.18

Nitrogen Burning

$$^{14}N(\alpha,\gamma)^{18}F(\beta^{+}\nu_{e})^{18}O(\alpha,\gamma)^{22}Ne$$

- ^{14}N is made as slowest reactant in CNO cycle
- It is made from initial metals, not as a primary product
- Depending on metallicity, the abundance can be come significant; it will be more important for more metal-rich stars.
- ^{14}N burning occurs at the onset – before – central helium burning and can have its own convective burning phase, take a few % of helium burning time.

Lecture 11 Problem Set

Please do calculations and provide results using cgs units.

Nuclear Reaction Kinematics

Based on the general formula for nuclear reactions,

$$\frac{\partial}{\partial t} Y_i = \sum_{\substack{\alpha_1, \alpha_2, \ldots \\ \beta_1, \beta_2, \ldots}} \lambda_{\alpha_1 \mathbf{1} + \alpha_2 \mathbf{2} + \ldots \rightarrow \beta_1 \mathbf{1} + \beta_2 \mathbf{2} + \ldots} \frac{\beta_i - \alpha_i}{\alpha_1! \alpha_2! \ldots} Y_1^{\alpha_1} Y_2^{\alpha_2} \ldots$$

1. **Write the equation for the change of hydrogen, $\frac{\partial}{\partial t} Y_{^1\mathrm{H}}$, in the reaction of the last step of the CNO-1 cycle for hydrogen bunring, $^{15}\mathrm{N} + {}^1\mathrm{H} \longmapsto {}^{12}\mathrm{C} + {}^4\mathrm{He}$ (simple binary reaction prototype).**

$$\frac{\partial}{\partial t} Y_{^1\mathrm{H}} = -\lambda_{^1\mathrm{H} + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^4\mathrm{He}} Y_{^1\mathrm{H}} Y_{^{15}\mathrm{N}}$$

2. **The system of equations for the changes $\frac{\partial}{\partial t} Y_i$ of $(i =)$ ^{1}H, ^{2}H, ^{3}He, and ^{4}He due to the pp1 chain for hydrogen burning. Assume a net production of 1 (one) ^{4}He nucleus from 4 (four) ^{1}H nuclei. Assume the reaction is in equilibrium (steady state).**

$$\frac{\partial}{\partial t} Y_{^1\mathrm{H}} = -\lambda_{2\,^1\mathrm{H} \rightarrow {}^2\mathrm{H}} \left(Y_{^1\mathrm{H}}\right)^2 - \lambda_{^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} + \lambda_{2\,^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,^1\mathrm{H}} \left(Y_{^3\mathrm{He}}\right)^2 \quad (1)$$

$$\frac{\partial}{\partial t} Y_{^2\mathrm{H}} = +\lambda_{2\,^1\mathrm{H} \rightarrow {}^2\mathrm{H}} \frac{1}{2} \left(Y_{^1\mathrm{H}}\right)^2 - \lambda_{^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} \quad (2)$$

$$\frac{\partial}{\partial t} Y_{^3\mathrm{He}} = +\lambda_{^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He}} Y_{^1\mathrm{H}} Y_{^2\mathrm{H}} - \lambda_{2\,^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,^1\mathrm{H}} \left(Y_{^3\mathrm{He}}\right)^2 \quad (3)$$

$$\frac{\partial}{\partial t} Y_{^4\mathrm{He}} = +\lambda_{2\,^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,^1\mathrm{H}} \frac{1}{2} \left(Y_{^3\mathrm{He}}\right)^2 \quad (4)$$

3. **Assuming steady state, what are the timely changes of ^{2}H and ^{3}He?**

$$\frac{\partial}{\partial t} Y_{^2\mathrm{H}} = 0 = \frac{\partial}{\partial t} Y_{^3\mathrm{He}}$$

4. **Express the abundance of ^{2}H in terms of that of ^{1}H and the relevant reaction rates λ_i.**
From Eq. 2, and $\frac{\partial}{\partial t} Y_{^2\mathrm{H}}$ we can solve for $Y_{^2\mathrm{H}}$:

$$Y_{^2\mathrm{H}} = \frac{Y_{^1\mathrm{H}}}{2} \frac{\lambda_{2\,^1\mathrm{H} \rightarrow {}^2\mathrm{H}}}{\lambda_{^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He}}}$$

5. **Add specific values for the lambda's.**
Assume a central temperature of sun about $1.6 \times 10^7\,\mathrm{K}$, a central density about $160\,\mathrm{g\,cm^{-3}}$, and that the sun has burnt half of its ^{1}H fuel by now.

Note: For a binary reaction we have $\lambda = N_\mathrm{A} \langle \sigma v \rangle \rho$
Obtain values for nuclear reaction rates from
`http://starlib.physics.unc.edu/RateLib.php`

Using
$\lambda_{2\,^1\mathrm{H} \rightarrow {}^2\mathrm{H}} = 1.044 \times 10^{-19} \times 160\,\mathrm{g\,mol^{-1}\,s^{-1}}$,
$\lambda_{^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He}} = 1.482 \times 10^{-2} \times 160\,\mathrm{g\,mol^{-1}\,s^{-1}}$,
$Y_{^1\mathrm{H}} = 0.35$, we obtain

$$Y_{^2\mathrm{H}} = \frac{0.35 \times 1.044 \times 10^{-19}}{2 \times 1.482 \times 10^{-2}} = 1.233 \times 10^{-18}$$