

- POLICY:
- Respect others
  - Always ask if you can't follow
  - help answer questions using your own words
  - Do not disturb class with non-class related business

# LECTURE I: EOS

20170227-1-

Reading (Kuro 2012)

§§ 4.1-4.3, 8.1, 13, 15, 16

optional § 14 (ionisation)

Goal:

- Being able to build stellar model
- Compute stellar evolution
- Understand nuclear burning phases
  - which occur in what stars
  - where and when

→ need to know relevant physics  
→ need to know simple analytic solutions, so we can check models

- build simple stellar model (Lane-Emden)
- use stellar evolution code to run our models

Q: what would you need to make a model of a star?

→ where do we start?

- UNDERSTAND physics governing evolution of stars

# Equation of State (EOS)

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What is EOS?

→ composition + independent variables

E.g.:  $T, \rho$  - intensive quantities

→ give  $P, E, \dots$

Q: how many and which variables do we need?

• What [constituents] do we have to consider?

- Gas: ideal, degenerate, relativistic

Q: what gases [Ions, electrons, ...]

→ composition

? [virt. particles]

- Radiation

But: change of composition due to

- ionisation

- nuclear reactions

→ strong, weak forces

Q: What does each do?

• chemical or nuclear equilibrium:

Q: Do they change EOS?

How important for EOS?

→ can be incorporated in EoS 20170227-3-

Q: what does this mean? How to do, when?

[A: change fast compared to external evolution]

→ EoS has many components...  
How to combine?

EXAMPLE: Total Pressure

Simple case:  $P_{TOT} = \sum P_i$   
partial pressures:

Q: is this always so [No]

Q: which pieces should/can we study separately?

How do we best divide things up?

Typical case (relevant under not too extreme conditions)

$$P = P_{RAD} + \underbrace{P_{ions} + P_{e^-}}_{P_{gas}}$$

Q: Do you know formulas for certain conditions?

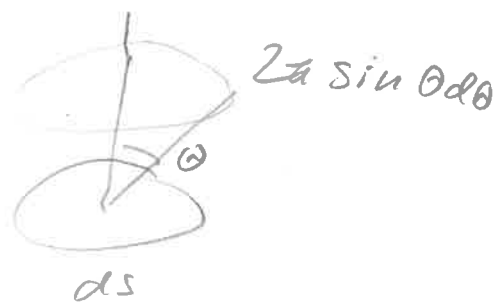
Similar for E:

$$E = E_{RAD} + \underbrace{E_{ions} + E_{e^-}}_{E_{gas}}$$

But let's derive in detail...

Pressure

$$\frac{\# \text{ particles}}{\text{Volume} \cdot dp}$$



$ds$  — surface element

- 1) fraction of particles at angle  $\theta$ :

$$\frac{2\pi \sin \theta d\theta}{4\pi} = \frac{1}{2} \sin \theta d\theta$$

$4\pi$  ← all possible directions

- 2) change in momentum (= momentum transfer)

$$dp = 2 \cos \theta \cdot p = 2 p_{\perp}$$

- 3) rate at which particles hit the surface:

$$\text{rate} = \cos \theta v(p) \cdot n(p) ds$$

$$\rightarrow P = \int_0^{\pi/2} 2 \cos \theta \cdot \frac{1}{2} \sin \theta \cos \theta d\theta \times \int_0^{\infty} n(p) v(p) p dp$$

$$P = \frac{1}{3} \int_0^{\infty} n(p) v(p) p dp$$

Q: why do we use  $p$  as variable for integration? [not  $v$ ] [consider specific cases]

## Energy density

Similar, but energy is scalar not vector  
 → no geometric factor from projection

$$u = \frac{1}{g} \int_0^{\infty} n(p) E(p) dp$$

energy of particle  
with momentum  $p$

specific internal  
energy

Q: what would be formula for  
 $E(p)$  for specific cases?

Q: and for  $n(p)$ ?

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## Build Your Own Star

Use [MESA.STAR.ORG](http://MESA.STAR.ORG)

[MESA.SOURCEFORGE.NET](http://MESA.SOURCEFORGE.NET)

Download, install, run test case