Stellar Structure

1) Mass coordinate

$V_{sphere} = \frac{4\pi}{3} r^3$

ADVANTAGE in CODES: IMPLICIT MASS conservation
→ m · Lagrange coordinate

$dV = 4\pi r^2 dr$

$dm = \rho \cdot dV = 4\pi r^2 \rho \, dr$

$$\boxed{\frac{dm}{dr} = 4\pi r^2 \rho}$$

use: $\frac{d}{dm} = \frac{1}{4\pi r^2 \rho} \frac{d}{dr}$ | coordinate transform

NB Sometime people use ... used to use ...

$q = \frac{m}{M}$ ← total mass

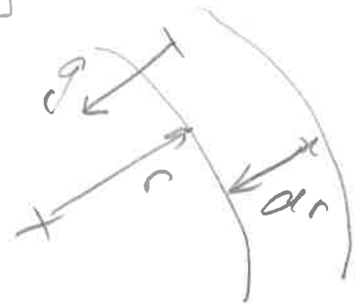
but: not Lagrangian if M is not constant!, e.g. due to mass loss by winds, binary star interaction
→ mass conservation is more difficult to formulate

! Assume structure & stellar properties are functions of t and m
→ coupled PDE system

2) Momentum equation 20170306-2-

[hydrostatic equilibrium]

$$g = -\frac{Gm(r)}{r^2} = -\frac{Gm}{r(m)^2}$$



force balance

$$-\frac{\partial P}{\partial r} + g\cdot\rho = 0 \Longleftrightarrow \text{no net acceleration}$$

$$\frac{\partial P}{\partial r} = g\rho = -\frac{Gm(r)}{r^2}\cdot\rho \quad \Big| \quad \frac{\partial}{\partial m} = \frac{1}{4\pi r^2\rho}\frac{\partial}{\partial r}$$

$$\boxed{\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}}$$

Q: Estimate P_c, T_c in terms of average ρ: $\bar\rho$
use ideal gas | and central ρ: ρ_c

... assume mean values

$P_0 = 0$ $\quad m \sim M/2$

$P_c = ?$ $\quad r \sim R/2$

$$P_c = \frac{16GM}{2\cdot 4\pi R^4} = \frac{2GM^2}{\pi R^4} = \frac{8}{3}\frac{G\bar\rho M}{R}$$

typical $\frac{\bar\rho}{\rho_c} \ll 1$

$$\bar\rho = \frac{3M}{4\pi R^3}$$

$$P_c = \frac{\mathcal{R}T_c\rho_c}{\mu}$$

$$\rightarrow T_c = \frac{P_c}{\rho_c}\frac{\mu}{\mathcal{R}} = \frac{P_c}{\rho_c}\frac{\bar\rho 4\pi R}{3M}$$

$$\simeq \frac{8}{3}\frac{\mu}{\mathcal{R}}\frac{GM}{R}\frac{\bar\rho}{\rho}$$

Hydrodynamic Version

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$$\frac{\partial P}{\partial r} = g\varrho - \varrho \frac{\partial^2 r}{\partial t^2}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{\partial^2 r^2}{\partial t^2}$$

w/o Pressure: FREE FALL (direct collapse)

$$\tau_{FF} \simeq \left(\frac{R}{g}\right)^{1/2} \simeq \frac{1}{2} \frac{1}{\sqrt{G\bar{\varrho}}}$$

[Q: compute solar value!

spherical collapse:

$$\tau_{ff} = \sqrt{\frac{3\pi}{32 G \bar{\varrho}(m)}}$$

[Q: derive $\ddot{r}(m)$ as function of $m, \bar{\varrho}, \varrho, \tau_{ff}$

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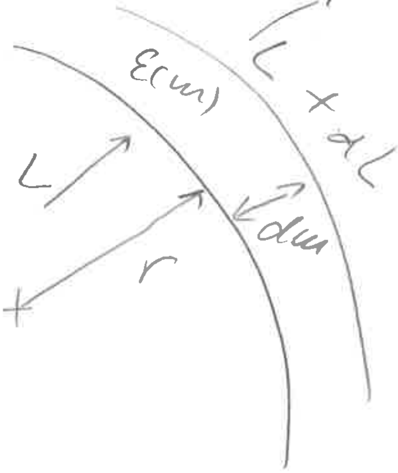
Energy Conservation



$$dL = 4\pi r^2 \rho\, \varepsilon\, dr = \varepsilon\, dm$$

$$\frac{\partial L}{\partial m} = \varepsilon = \varepsilon_{nuc} + \varepsilon_{\nu} + \varepsilon_g$$

where gravitational energy is given by

$$\varepsilon_g = -T\frac{\partial s}{\partial t} = -c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t}$$

[specific volume $v = 1/\rho$]

here we used $\quad dq = c_P\, dT - \frac{\delta}{\rho}\, dP$

$$\delta = \left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P$$

[and you often also find

$$\alpha := \left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T$$

(we will use later)]

[Q: what does this quantity δ actually mean?]

Estimate of Stellar Energy 20170307-5-

Kelvin Helmholtz time scale

$$\tau_{KH} = \frac{E_G}{L} \simeq \frac{2E_i}{L}$$

internal energy

virial theorem

HW Read up on it

$$|E_G| \simeq \frac{G\bar{m}^2}{\bar{r}} \simeq \frac{GM^2}{2R}$$

$$\rightarrow \tau_{KH} \simeq \frac{GM^2}{2RL}$$

Sun $\sim 1.6 \times 10^7\ yr$