Energy TRANSPORT

Concept: mean free path of photon

$l_{ph} = \frac{1}{\rho \cdot \kappa}$ typical for sun: 2cm

Diffusive flux: $j = -D \nabla n$

- $D = \frac{1}{3} v \cdot l_{ph}$ $(v = c)$
- ∇n? use energy density for n:

$$U = aT^4$$

$$\rightarrow \frac{\partial U}{\partial r} = 4aT^3 \frac{\partial T}{\partial r}$$

$$\rightarrow \text{diffusive flux } j \rightarrow F = -\underbrace{\frac{4ac}{3} \frac{T^3}{\kappa \rho}}_{} \frac{\partial T}{\partial r}$$

$$=: -K_{RAD} \cdot \frac{\partial T}{\partial r}$$

use: $l(m) = 4\pi r^2(m) F$ luminosity

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi a c r^2} \frac{\kappa \rho l}{T^3}$$

$r \rightarrow m$, i.e. $\frac{\partial}{\partial r} = 4\pi r^2 \rho \frac{\partial}{\partial m}$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 a c} \frac{\kappa l}{r^4 T^3}$$

in astro: use temperature gradient $\frac{\partial T}{\partial P}$

$$\frac{\partial T/\partial m}{\partial P/\partial m} = \frac{3}{16\pi a c G} \times \frac{\kappa l}{m T^3}$$

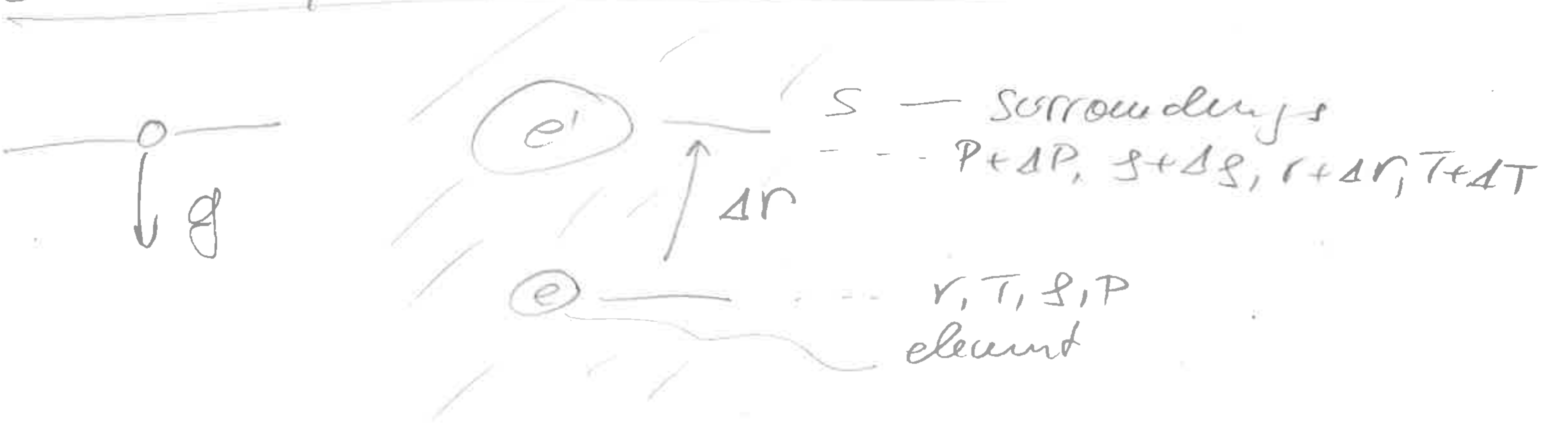
Def: $\nabla_{rad} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_{RAD} = \frac{3}{16\pi a c G} \frac{\kappa l P}{m T^4}$

$$\rightarrow \quad \frac{\partial T}{\partial m} = \frac{G m T}{4\pi P r^4} \nabla_{rad}$$

NOTE: Later we will see that this formalism can also be used to describe energy transport in other regimes!

Q: what other mode of energy transport might there be?

Local Dynamic (IN) STABILITY 20170308-3-



Stability analysis procedure:

! Dynamic stability analysis
→ No heat exchange ⟷ consider adiabatic displacement of eddy in stratified surrounding

then assess: "Is it BUOYANT?"

Condition for buoyancy: $\boxed{\rho_e' > \rho_s}$

here we assume:
- Pressure equilibrium
- No composition exchange
- No nuclear reactions

Let's look at small changes:

$$\left.\frac{d\rho}{dr}\right|_e > \left.\frac{d\rho}{dr}\right|_s$$

FOR STABILITY

This is tricky: $\frac{d\rho}{dr} < 0$ &

$$\left|\left.\frac{d\rho}{dr}\right|_e\right| < \left|\left.\frac{d\rho}{dr}\right|_s\right|$$

Q: what do we next?

General EOS

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for now: assume ideal gas with radiation
we can write: $P = aT^4 + \mathcal{R}T\rho/\mu$

$$\frac{d\rho}{\rho} = \alpha \frac{dP}{P} - \delta \cdot \frac{dT}{T} + \varphi \frac{d\mu}{\mu}$$

with: $\alpha := \left.\frac{d\ln\rho}{d\ln P}\right)_{T,\mu}$; $\delta := \left.\frac{-d\ln\rho}{d\ln T}\right)_{P,\mu}$; $\varphi = \left.\frac{d\ln\rho}{d\ln\mu}\right)_{P,T}$

Q: what do these quantities mean?

Put in:

$$\left.\frac{\alpha}{P}\frac{dP}{dr}\right)_e - \left.\frac{\delta}{T}\frac{dT}{dr}\right)_e + \left.\frac{\varphi}{\mu}\frac{d\mu}{dr}\right)_e > \left.\frac{\alpha}{P}\frac{dP}{dr}\right)_s - \left.\frac{\delta}{T}\frac{dT}{dr}\right)_s + \left.\frac{\varphi}{\mu}\frac{d\mu}{dr}\right)_s$$

cancel; no comp exchange

Pressure equilibrium: $\longrightarrow$ P-terms cancel

Def Pressure Scale Height:

$$H_P := -\frac{dr}{d\ln P} = -P\frac{dr}{dP} = -\frac{P}{g\rho} > 0$$

multiply by H_P:

$$\delta \left.\frac{d\ln T}{d\ln P}\right)_e > \delta \left.\frac{d\ln T}{d\ln P}\right)_s - \varphi \left.\frac{d\ln\mu}{d\ln P}\right)_s$$

DEF: $\nabla := \left.\frac{d\ln T}{d\ln P}\right)_s$; $\nabla_e = \left.\frac{d\ln T}{d\ln P}\right)_e$; $\nabla_\mu := \left.\frac{d\ln\mu}{d\ln P}\right)_s$

symbol, not quadrat

$$\longrightarrow \quad \nabla < \nabla_e + \frac{\varphi}{\delta}\nabla_\mu$$

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Specifically:

- $\nabla = \nabla_{rad}$ for stable structure
- $\nabla_e = \nabla_{ad}$ for adiabatic displacement

  → good approximation in convective regions

$$\boxed{\nabla_{RAD} < \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu}$$ Ledoux criterion for stability against convection

W/o composition gradients: $\boxed{\nabla_\mu = 0}$ we obtain:

$$\boxed{\nabla_{rad} < \nabla_{ad}}$$ Schwarzschild criterion for stability

Reasoning for use of Schwarzschild:

- once you mix stuff, composition gradients disappear irreversibly

→ in that sense, Ledoux is not "reversible"

→ dependence on initial conditions, unstable

→ once mixed, Schwarzschild applies (or is equivalent to Ledoux)