

Homework Set 1

Due: April 30, 2013, *before class*

1. Energy Generation

- (a) Compute the specific energy generation rate of the Sun as a whole.

$$\epsilon_{\text{sun}} = \frac{1 L_{\odot}}{1 M_{\odot}} = \frac{3.83 \times 10^{33} \text{ erg s}^{-1}}{1.9891 \times 10^{33} \text{ g}} = 1.93 \text{ erg g}^{-1} \text{ s}^{-1}$$

- (b) Assume a human of weight 100 kg has a “luminosity” of 100 W. Compute the specific energy generation rate of a this human.

$$\epsilon_{\text{human}} = \frac{10^9 \text{ erg s}^{-1}}{10^5 \text{ g}} = 10^4 \text{ erg g}^{-1} \text{ s}^{-1}$$

- (c) Compare the results from (a) and (b).

The specific energy generation of a human is about 5,000 times that of the sun.

- (d) Modern microprocessors have now reached a “gate width” of 25 nm. Assume this corresponds to the thickness of the “active” layer that contains microprocessors and density of silicon of 2.33 g/cm³. The typical die size is about 100 mm² and they have a power up about 100 W. What is the specific energy generation rate of the active layer?

$$\epsilon_{\text{CPU}} = \frac{10^9 \text{ erg s}^{-1}}{2.5 \times 10^{-6} \text{ cm} \times 1 \text{ cm}^2 \times 2.33 \text{ g cm}^{-3}} = 1.72 \times 10^{14} \text{ erg g}^{-1} \text{ s}^{-1}$$

- (e) How long does it take for the “active” layer of the CPU to release as much energy as their rest mass?

$$\tau = \frac{c^2}{\epsilon_{\text{CPU}}} = 5.2 \times 10^6 \text{ s} = 1,440 \text{ h} = 60 \text{ d} = 0.16 \text{ yr}$$

- (f) What happens when you run your computer that long – having converted the rest mass of the “active” layer into energy?

Well, the CPU does not dissipate internal energy or its own rest mass, but rather electricity supplied from the outside. But, yes, that electricity comes from rest mass somewhere else, e.g., stored in nuclear or chemical form, or in gravitational potential energy.

- (g) What is the specific energy generation rate corresponding to an element of mass radiating away its entire rest mass in 1 s?

$$\epsilon = \frac{c^2}{1 \text{ s}} = 8.91 \times 10^{20} \text{ erg g}^{-1} \text{ s}^{-1}$$

- (h) Assume a characteristic chemical energy content of 10 eV per nucleon, and a characteristic nuclear energy content of 10 MeV per nucleon.

Compute the energy content (supply) of the sun for each of these assumptions. How long could the sun shine at its current luminosity from each of these energy sources?

$$E_{\text{chemical},\odot} = 10 \text{ eV} \times N_A \times M_{\odot} = 1.92 \times 10^{46} \text{ erg}$$

$$E_{\text{nuclear},\odot} = 10 \text{ MeV} \times N_A \times M_{\odot} = 1.92 \times 10^{52} \text{ erg}$$

$$\tau_{\text{chemical},\odot} = E_{\text{chemical},\odot} / L_{\odot} = 5.01 \times 10^{12} \text{ sec} = 1.59 \times 10^5 \text{ yr}$$

$$\tau_{\text{nuclear},\odot} = E_{\text{nuclear},\odot} / L_{\odot} = 5.01 \times 10^{18} \text{ sec} = 1.59 \times 10^{11} \text{ yr}$$