Homework Set 5

Due: May 25, 2013, *before class*

1. **Stellar Model with Thermonuclear Burning**

Based on the Lane Emden Equation solver developed for Homework Set 3, construct stellar models that include nuclear burning.

Assume a composition that is a mixture of 70 % hydrogen (^{1}H), 28 % helium (^{4}He) and 2 % of nitrogen (^{14}N), by mass fraction.

(a) Compute a model with $n = 1.5$ and $M = 1\,\mathrm{M}_\odot$. Assume the star generates its energy according to the pp chains. Use formula 18.63 from Kippenhahn & Weigert (1990). For simplicity, assume $\psi = 1$ and $f_{11} = 2$. Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

Find central density such that the luminosity of star equal the solar luminosity. What is the radius of the star? Why does it deviate from that of the sun?

First we compute the mean molecular weight,

$$\mu = \left(\sum_i \frac{X_i\,(Z_i+1)}{A_i}\right)^{-1},$$

for which we find about $\mu = 0.617$. We then compute density for each zone using

$$\rho(\xi) = \rho_\mathrm{c}\,\Theta(\xi)^n$$

and obtain pressure for each zone from

$$P = K\rho^{(n+1)/n}$$

where we obtained K in the usual way,

$$K = \alpha^2 \frac{4\pi G \rho_\mathrm{c}^{(n-1)/n}}{n+1}$$

with

$$\alpha = \sqrt[3]{-\frac{M}{4\pi\rho_\mathrm{c}\xi_1^2\,\frac{\mathrm{d}\Theta}{\mathrm{d}\xi}\Big|_{\xi_1}}}$$

.

Then we solve the equation for ideal gas pressure,

$$P = \frac{\mathcal{R}T\rho}{\mu}$$

for temperature,

$$T = \frac{P\mu}{\mathcal{R}T}$$

(or, more accurately, use ideal gas with radiation), and can compute the energy generation rate in units of erg/g/s for the approximation formula in the book.

To integrate the total energy generation rate, we first compute the mass of each zone by multiplying its volume by the average density,

$$\Delta m = \frac{1}{2}\,(\rho(\xi) + \rho(\xi+\Delta\xi))$$

and then multiply this by the average energy generation rate to obtain the contribution to luminosity,

$$\Delta L = \Delta m\,\frac{1}{2}\,(\varepsilon(\xi) + \varepsilon(\xi+\Delta\xi))$$

. We now vary central density until the total luminosity equals the solar luminosity.

For the sun I find a match for $\rho_c \approx 50.4\,\mathrm{g\,cm^{-3}}$ and for this model the radius is $3.82 \times 10^{10}\,\mathrm{cm} = 0.547\,\mathrm{R_\odot}$. As you know, this is only a very crude approximation for structure of the present sun, there is contributions from the CNO cycle, we did only a crude fixed guess for g_{11} in the reaction rate formula, but even more important the, the sun has already evolved (so hydrogen mass fraction in the center has dropped by about half, and has an outer convection zone, etc.)

As a note, this model has $\beta = 0.999$ and hence not taking into account radiation pressure is OK, however, the gas is not just an ideal gas: electron degeneracy starts playing a role for the sun. Score: **6**

(b) Compute a model with $n = 3$ and $M = 100\,\mathrm{M_\odot}$. Assume the star generates its energy according to the CNO cycle. Use formula 18.65 from Kippenhahn & Weigert (1990). Assume the star is in thermal and hydrostatic equilibrium, i.e., nuclear energy generation balances luminosity from the surface.

What is the luminosity and radius of the star for central temperature of $2 \times 10^7\,\mathrm{K}$, $2.5 \times 10^7\,\mathrm{K}$, $3 \times 10^7\,\mathrm{K}$, and $3.5 \times 10^7\,\mathrm{K}$?

Similar to above, but now we have to use for sure an ideal gas with radiation,

$$P = \frac{a}{3}T^4 + \frac{\mathcal{R}T\rho}{\mu}$$

, and solve for temperature for each zone from that,

$$\tilde{V} = \left(\frac{3\mathcal{R}\rho}{4\mu a}\right)^2, V = \sqrt[3]{\tilde{V} + \sqrt{\tilde{V}^2 + (P/a)^3}}, \tilde{T} = 2\left(V - \frac{P}{aV}\right), T = \frac{1}{2}\left(\sqrt{\left(\frac{6\mathcal{R}\rho}{\mu a\sqrt{\tilde{T}}}\right) - \tilde{T}} - \sqrt{\tilde{T}}\right).$$

Using the same approximations and procedure as above but for energy generation using the formula for the CNO cycle, we find $\beta = 0.558$ and

T_c ($10^7\,\mathrm{K}$)	ρ_c ($\mathrm{g\,cm^{-3}}$)	L ($\mathrm{L_\odot}$)	R ($\mathrm{R_\odot}$)	T_{eff} (kK)
2.0	0.192	2.11	34.0	1.19
2.5	0.377	2.27×10^2	27.1	4.30
3.0	0.65	8.00×10^3	22.6	11.5
3.5	1.035	1.45×10^5	19.4	25.6
4.0	1.55	1.63×10^6	16.9	50.1

Score: **6**