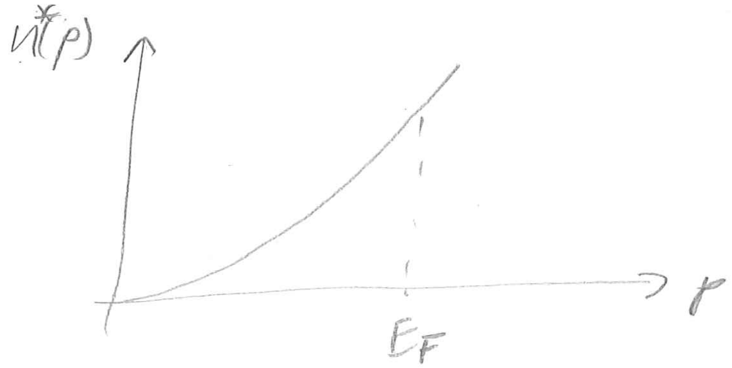
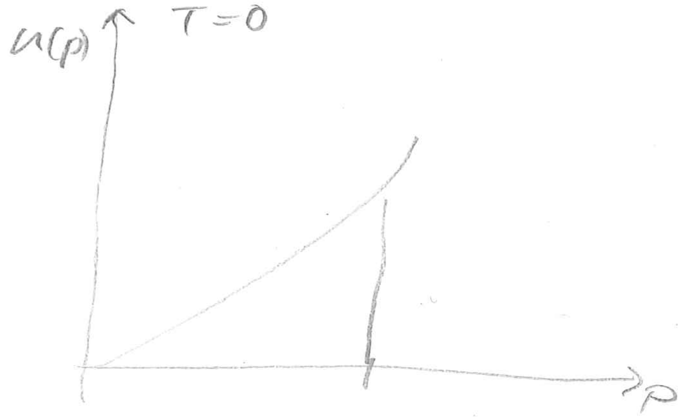
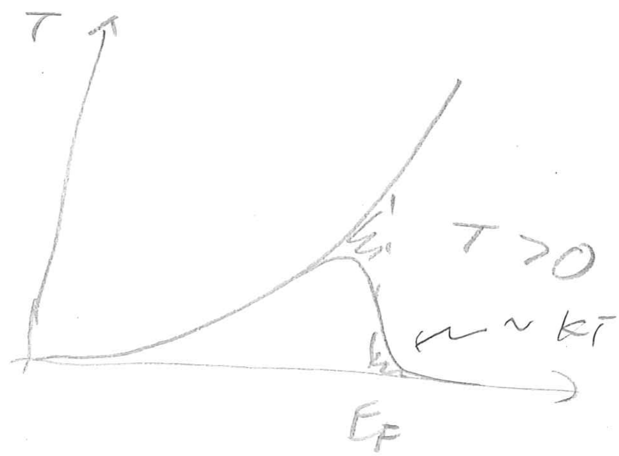
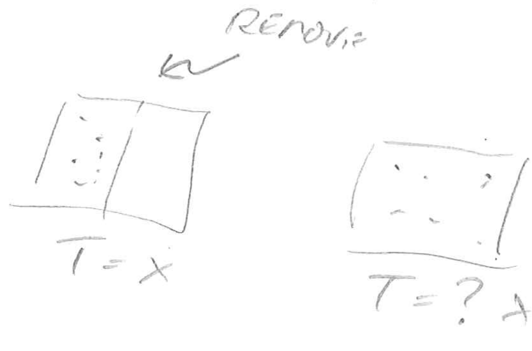
Q DEGENERATE GAS



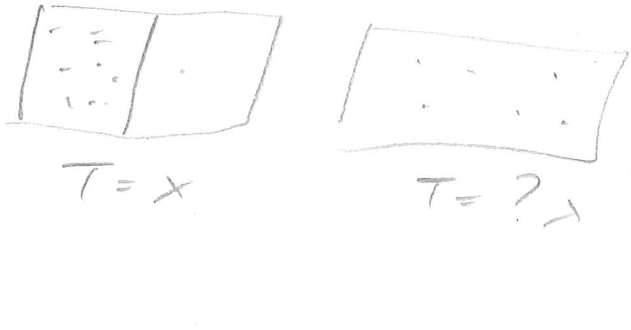




FOR IDEAL GAS



DEGENERATE GAS



STELLAR STRUCTURE

1) mass COORDINATE!

(mass conservation implicit)

$V_{SPHERE} = \frac{4\pi}{3} R^3$

$\rightarrow dV = 4\pi r^2 dr$

$dm = \rho dV = 4\pi r^2 \rho dr$ $\boxed{\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho}}$

m: LAGRANGIAN COORDINATE

use $\frac{\partial}{\partial m} = \frac{1}{4\pi r^2 \rho} \frac{\partial}{\partial r}$

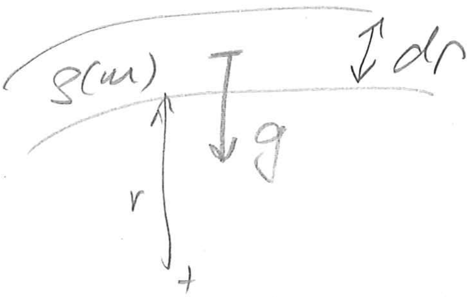
Sometimes people use (d to m)

$q = \frac{m}{M}$ M: total mass of star

$\rightarrow$ not LAGRANGIAN ... conservation more difficult to formulate!

2) MOMENTUM CONSERVATION

Hydrostatic equilibrium



$$g = -\frac{Gm}{r^2}$$

$$\frac{\partial P}{\partial r} + \rho g = 0 \quad \text{(no net acceleration)}$$

$$\frac{\partial P}{\partial r} = -g\rho = -\frac{Gm}{r^2}\rho \qquad \Big| \; \frac{\partial}{\partial m} = \frac{1}{4\pi r^2 \rho}\frac{\partial}{\partial r}$$

$$\rightarrow \boxed{\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}}$$

ESTIMATE P_c, (ρ_c), T_c ← IDEAL GAS in terms of average ρ

↑ MEANS ↖ AVERAGE ρ

① $P_0 = 0$

$P_c \ldots ?$

$m = \frac{M}{2}$

$r \sim R/2$

$$P_c \approx \frac{16\,GM}{2\cdot 4\pi R} = \frac{2GM}{\pi R^4}$$

② $\boxed{\bar{\rho} = \frac{3M}{4\pi R^3}}$

$$P_c = \frac{\Re T_c \rho_c}{\mu}$$

$$T_c = \frac{P_c}{\rho_c}\frac{\mu}{\Re} = P_c \frac{\mu}{\Re}\frac{\bar{\rho}}{\rho_c}\frac{4\pi R^3}{3M}$$

$$\approx \frac{8}{3}\frac{\mu}{\Re}\frac{GM}{R}\frac{\bar{\rho}}{\rho_c}$$

typical $\frac{\bar{\rho}}{\rho_c} \ll 1$

hydrodynamic

$$\frac{\partial P}{\partial r} = -g\rho - \frac{\partial^2 r}{\partial t^2}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

w/o PRESSURE: free fall

$$\tau_{ff} \simeq \left(\frac{R}{g}\right)^{1/2} \simeq \frac{1}{2}\frac{1}{\sqrt{G\rho}}$$

KELVIN HELMHOLTZ TIME SCALE

$$\tau_{KH} := \frac{E_G}{L} \simeq \frac{E_I}{L}$$

$$|E_G| \simeq \frac{G\bar{m}^2}{\bar{r}} \simeq \frac{GM^2}{2R}$$

$$\boxed{\rightarrow \tau_{KH} \simeq \frac{GM^2}{2RL}}$$

Sun

$\sim 1.6 \times 10^7$ yr