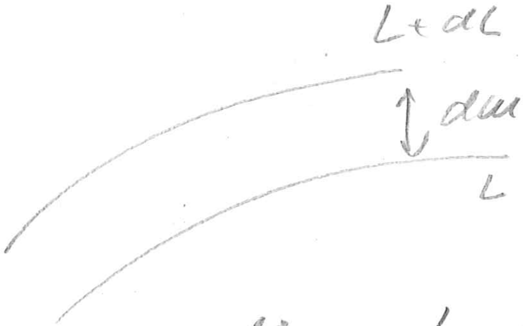
ENERGY CONSERVATION



$$dL = 4\pi r^2 \rho\, \varepsilon\, dr = \varepsilon\, dm$$

$$\frac{\partial L}{\partial m} = \varepsilon \quad \leftarrow \text{specific}$$

$$\varepsilon = \varepsilon_{nuc} - \varepsilon_\nu + \varepsilon_g$$

$$\varepsilon_g = -c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} = -\frac{\partial u}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} = -T\frac{\partial s}{\partial t}$$

$$c_P = \left.\frac{\partial q}{\partial T}\right|_P = \left.\frac{\partial u}{\partial T}\right|_P + P\left.\frac{\partial v}{\partial T}\right|_P \qquad v = \frac{1}{\rho}$$

$$\alpha = \left.\frac{\partial \ln \rho}{\partial \ln P}\right|_T$$

$$\delta = \left.\frac{\partial \ln \rho}{\partial \ln T}\right|_P$$

$$\frac{\partial L}{\partial m} = \varepsilon_n - \varepsilon_\nu - T\frac{\partial s}{\partial t} \qquad dq = -T\frac{\partial s}{\partial t}$$

Energy transport

mean free path

$$l_{ph} = \frac{1}{\kappa \rho}$$ typical for sun 2cm

$$j = -D\, \nabla n \quad , \quad D = \frac{1}{3} v\, l_p$$

Energy density $U = aT^4$

$$\frac{\partial U}{\partial r} = 4aT^3 \frac{\partial T}{\partial r}$$

$$\rightarrow F = -\underbrace{\frac{4ac}{3} \frac{T^3}{\kappa \rho}}_{K_{rad}} \frac{\partial T}{\partial r} \qquad \Big| \; l = 4\pi r^2 F$$

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi a c r^2 T^3} \kappa \rho l$$

$$\frac{\partial T}{\partial m} = -\frac{3}{64\pi^2 a c} \frac{\kappa l}{r^4 T}$$

Use Temperature gradient $\frac{\partial T}{\partial P}$ ...

$$\left(\frac{\partial T/\partial m}{\partial P/\partial m}\right) = \frac{3}{16\pi a c G}\frac{\kappa l}{m T^3}$$

DEF. $$\nabla_{rad} = \left.\frac{\partial \ln T}{\partial \ln P}\right|_{rad} = \frac{3}{16\pi a c G}\frac{\kappa l P}{m T^4}$$

$$\rightarrow \boxed{\frac{\partial T}{\partial m} = -\frac{G m T}{4\pi r^4 P}\nabla_{(rad)}}$$

Later we will see that this can also be used in other regimes

convection!