LOCAL DYNAMIC (IN)STABILITY

DISPLACE EDDY ADIABATICALLY : IS IT BOOYANT ?
($\rightarrow$ no heat exchange) $\rightarrow$ $\boxed{\rho_e' < \rho_s}$

look @ small changes:

$$\left(\frac{d\rho}{dr}\right)_e > \left(\frac{d\rho}{dr}\right)_s$$

NOTE $\frac{d\rho}{dr} < 0$

FOR STABILITY !

general EOS

$$\frac{d\rho}{\rho} = \alpha \frac{dP}{P} - \delta \frac{dT}{T} + \varphi \frac{d\mu}{\mu}$$

$$\alpha := \left.\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu} ; \quad \delta := -\left.\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$$

$$\varphi = \left.\frac{\partial \ln \rho}{\partial \ln \mu}\right|_{P,T}$$

Q: How would you "name" these quantities

ADIABATIC DISPLACEMENT: ALSO $d\mu_e = 0$

PUT INTO relation

$$\frac{\alpha}{P}\frac{dP}{dr}\Big|_e - \frac{\delta}{T}\frac{dT}{dr}\Big|_e + \overbrace{\frac{\varphi}{\mu}\frac{d\mu}{dr}\Big|_e}^{\sim 0} >$$

PRESSURE EQUIL. $\rightarrow$ $\shortparallel$

$$\frac{\alpha}{P}\frac{dP}{dr}\Big|_s - \frac{\delta}{T}\frac{dT}{dr}\Big|_s + \frac{\varphi}{\mu}\frac{d\mu}{dr}\Big|_s$$

introduce <u>PRESSURE SCALE HIGHT</u>

$$H_P := -\frac{dr}{d\ln P} = -P\frac{dr}{dP} \overset{\text{hydrostatic equilibrium}}{=} -\frac{P}{g\rho}$$

<u>multiply by H_P</u>

$$\delta\frac{d\ln T}{d\ln P}\Big|_e > \delta\frac{d\ln T}{d\ln P}\Big|_s - \varphi\frac{d\ln\mu}{d\ln P}\Big|_s$$

<u>DEF</u> $\nabla := \frac{d\ln T}{d\ln P}\Big|_s$; $\nabla_e := \frac{d\ln T}{d\ln P}\Big|_e$; $\nabla_\mu := \frac{d\ln\mu}{d\ln P}\Big|_s$

SYMBOL NOT OPERATOR!

$$\rightarrow \nabla < \nabla_e + \frac{\varphi}{\delta}\nabla_\mu$$

Specifically:

$\nabla = \nabla_{rad}$ for stable(?) stratification

$\nabla_e = \nabla_{ad}$ (adiabatic displacement
→ good approx in
convective regions as well)

$$\boxed{\nabla_{rad} < \nabla_{ad} + \frac{\varphi}{\delta} \nabla_\mu}$$

Ledoux
CRITERION
for stability against
convection

w/o composition gradients: $\boxed{\nabla_\mu \equiv 0}$

$$\boxed{\nabla_{rad} < \nabla_{ad}}$$

SCHWARZSCHILD CRITERION
(for stability)

Semiconvection

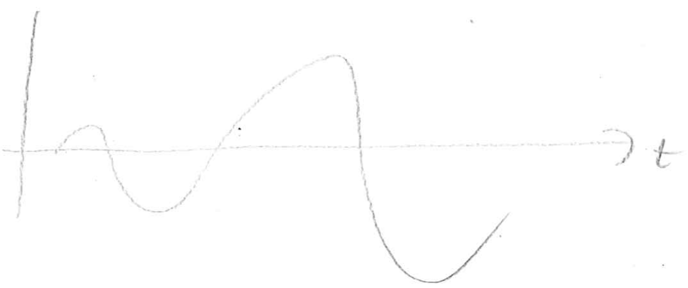
STABILIZED BY COMPOSITION GRADIENT

→ OSCILLATORY INSTABILITY

dq (because hotter

← now colder ⇒ ↓

dq (because colder)



- eventually: enough energy in oscillation to do full turnover

→ LAYER FORMATION

→ effective transport of heat, but composition constant into layers (mean free path of photons much larger than that of ions)

$$\tau_{DIFF} = l^2/D$$

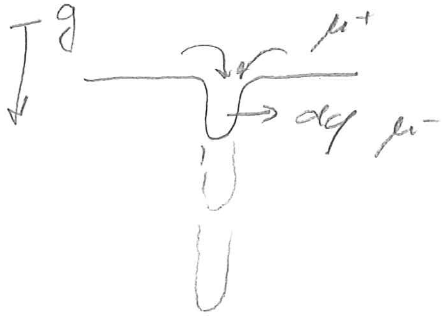
- on long time scale: merging of layers!

Thermohaline convection

⟺ salt finger instability
observed in ocean

hot heavy above light cool water

ocean heated on top, water evaporates, but salt is left behind → larger μ, but warmer
→ cooling at interface to cold water below → higher density → sinks



thin "fingers" cool easier

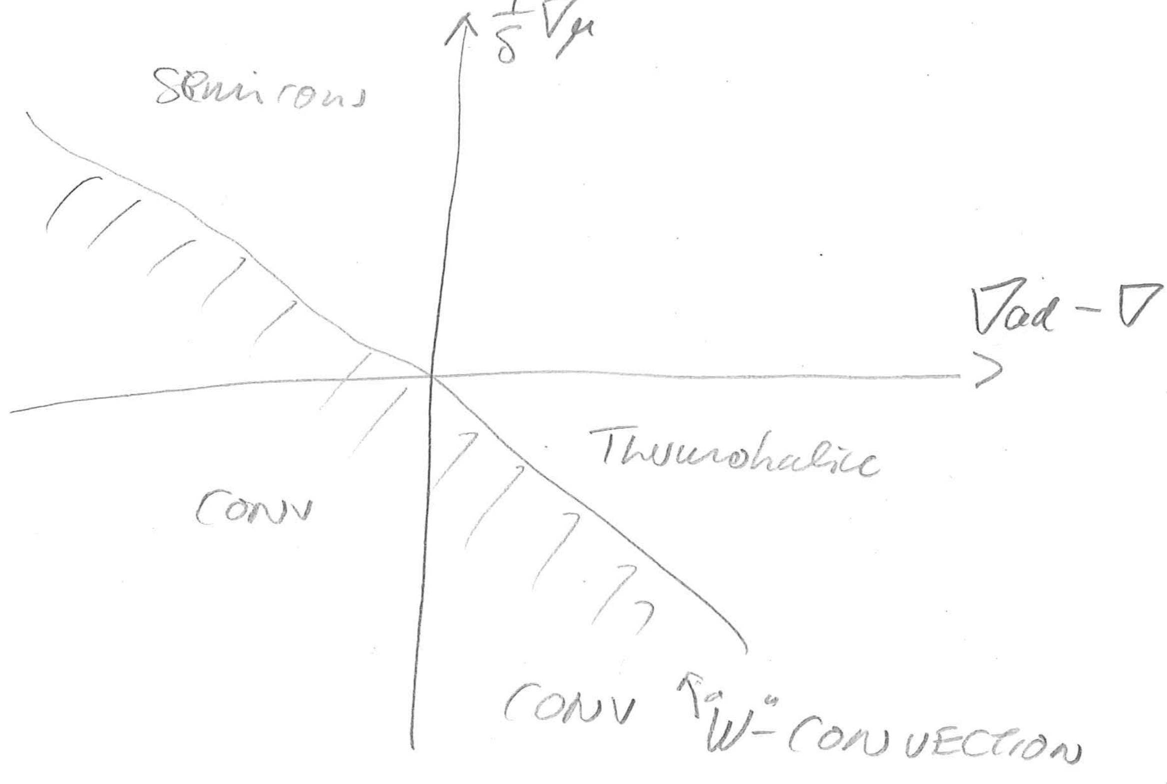
A CASE IN STARS:

- ACCRETION IN BINARY STAR SYSTEM
- ^3He burning

$$^3He + {}^3He \longrightarrow p + p + {}^4He$$

(compare μ on both sides)

What will happen?



REGIMES OF STABILITY

SC $\quad \nabla_{rad} < \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu$ + $\nabla_{rad} > \nabla_{ad}$ | $\nabla_{ad} < \nabla_{rad} < \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu$

TH $\quad \nabla_{rad} > \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu$ + $\frac{\varphi}{\delta}\nabla_\mu < 0$ | $\nabla_{ad} - \nabla_{rad} < \frac{\varphi}{\delta}\nabla_\mu < 0$

Q: DRAW SCHWARZSCHILD CRITERION IN DIAGRAM