Homework Set 3

Due: Mar 27, 2014, *before class*

1. **Simple Stellar Models**

   Write a program to solve the Lane-Emden Equation.

   Suggested steps are:

   (a) Convert Lane-Emden Equation into a system of 1st order linear differential equations.

   (b) find inner boundary condition as needed for integration; formally the term is 0/0, so you have to find the right limit by Taylor expansion or other method.

   $$\lim_{\xi \to 0} \frac{\mathrm{d}^2\theta\left(\xi\right)}{\mathrm{d}\xi^2} = \ldots$$

   (c) Write integration program using finite steps. Don't just use pre-compiled solver package. It is reasonable possible to do this just with a spreadsheet like Goolge Documents.

   (d) Find value of ξ where θ crosses 0. In the lecture we called this value ξ_1. If you use finite steps you could use interpolation to refine the estimate. Also obtain

   $$\left.\frac{\mathrm{d}\theta\left(\xi\right)}{\mathrm{d}\xi}\right|_{\xi_1} .$$

   Show and document you solution steps and program (submit program or formulas used in spreadsheet).

   Tasks:

   (a) Compute a model for $n = 1.5$, $M = 1\,\mathrm{M}_\odot$, central density $\rho_c = 10\,\mathrm{g\,cm^{-3}}$. Use ideal gas with $\mu = 1.3$. **What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star?**

   (b) Compute a model for $n = 1.5$, $M = 10\,\mathrm{M}_\odot$, $\rho_c = 1\,\mathrm{g\,cm^{-3}}$, and a mixture of 70 % hydrogen (^{1}H), 28 % helium (^{4}He) and 2 % of nitrogen (^{14}N), all percentages by mass fraction. Use ideal gas *with radiation*. Assume the gas is fully ionized.

   What is the radius of the star, what are central pressure, P_c and central temperature, T_c, of the star? What is fraction of gas to total pressure,

   $$\beta = \frac{P_{\mathrm{gas}}}{P}$$

   at the center of the star?